VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

ANS

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Victorian Certificate of Education
2021

STUDENT NUMBER									Letter

SPECIALIST MATHEMATICS

Written examination 1

Friday 5 November 2021

Reading time: 9.00 am to 9.15 am (15 minutes)
Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 13 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

Question 1 (4 marks)

The net force acting on a body of mass 10 kg is $\underset{\sim}{F} = 5\underset{\sim}{i} + 12\underset{\sim}{j}$ newtons.

a. Find the acceleration of the body in ms^{-2}. 1 mark

$\underset{\sim}{R} = M\underset{\sim}{a}$

$5\underset{\sim}{i} + 12\underset{\sim}{j} = 10\underset{\sim}{a}$

$a = \frac{1}{2}\underset{\sim}{i} + \frac{6}{5}\underset{\sim}{j}$

b. The initial velocity of the body is $-3\underset{\sim}{j}$ ms^{-1}.

Find the velocity of the body, in ms^{-1}, at any time t seconds. 2 marks

$\underset{\sim}{v} = \int (\frac{1}{2}\underset{\sim}{i} + \frac{6}{5}\underset{\sim}{j})\,dt$

$\underset{\sim}{v} = \frac{1}{2}t\underset{\sim}{i} + \frac{6}{5}t\underset{\sim}{j} + C$

$t = 0 \quad v = -3\underset{\sim}{j}$

$-3\underset{\sim}{j} = 0\underset{\sim}{i} + 0\underset{\sim}{j} + C$

$C = -3\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{1}{2}t\underset{\sim}{i} + \frac{6}{5}t\underset{\sim}{j} - 3\underset{\sim}{j}$

$\underset{\sim}{v} = \frac{1}{2}t\underset{\sim}{i} + (\frac{6}{5}t - 3)\underset{\sim}{j}$

c. Find the momentum of the body, in kg ms^{-1}, when $t = 2$ seconds. 1 mark

$\underset{\sim}{p} = m\underset{\sim}{v}$

$= 10 \times [\underset{\sim}{i} + (\frac{12}{5} - 3)\underset{\sim}{j}]$

$= 10 \times [\underset{\sim}{i} + (\frac{12}{5} - \frac{15}{5})\underset{\sim}{j}]$

$= 10 \times [\underset{\sim}{i} - \frac{3}{5}\underset{\sim}{j}]$

$= 10\underset{\sim}{i} - 6\underset{\sim}{j}$

DO NOT WRITE IN THIS AREA

Question 2 (3 marks)

Evaluate $\int_0^1 \frac{2x+1}{x^2+1}\,dx$.

$= \int_0^1 \frac{2x}{x^2+1}\,dx + \int_0^1 \frac{1}{x^2+1}\,dx$

Let $u = x^2+1$

$\frac{du}{dx} = 2x.$

$du = 2x\,dx.$

$x=0 \quad u=1$

$x=1 \quad u=2..$

$= \int_1^2 \frac{1}{u}\,du + \int_0^1 \frac{1}{x^2+1}\,dx$

$= \left[\log_e(u)\right]_1^2 + \left[\tan^{-1}(x)\right]_0^1$

$= \left[\log_e(2) - \log_e(1)\right] + \left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= \log_e(2) - 0 + \frac{\pi}{4} - 0.$

$= \log_e(2) + \frac{\pi}{4}.$

Question 3 (5 marks)

A company produces a particular type of light globe called Shiny. The company claims that the lifetime of these globes is normally distributed with a mean of 200 weeks and it is known that the standard deviation of the lifetime of Shiny globes is 10 weeks. Customers have complained, saying Shiny globes were lasting less than the claimed 200 weeks. It was decided to investigate the complaints. A random sample of 36 Shiny globes was tested and it was found that the mean lifetime of the sample was 195 weeks.

Use $\Pr(-1.96 < Z < 1.96) = 0.95$ and $\Pr(-3 < Z < 3) = 0.9973$ to answer the following questions.

a. Write down the null and alternative hypotheses for the one-tailed test that was conducted to investigate the complaints. 1 mark

$p = \Pr(\bar{X} \le \bar{x} \mid \mu)$

$= \Pr\left(Z \le \frac{\bar{x}-\mu}{sd(\bar{X})}\right)$

$H_0: \mu = 200$

$H_1: \mu < 200.$

b. i. Determine the p value, correct to three decimal places, for the test. 2 marks

$E(\bar{X}) = \mu = 200$

$sd(\bar{X}) = \frac{10}{\sqrt{36}} = \frac{10}{6} = \frac{5}{3}.$

$p = \Pr(\bar{X} \le 195 \mid \mu = 200).$

$= \Pr\left(Z \le \frac{195-200}{5/3}\right)$

$= \Pr\left(Z \le -5 \times \frac{3}{5}\right)$

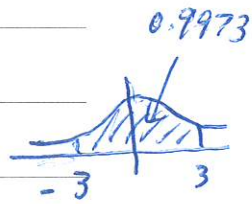
$p = \Pr(Z \le -3)$

$= \frac{1-0.9973}{2}$

$= \frac{0.0027}{2}$

$= 0.00135$

$= 0.001$



ii. What should the company be told if the test was carried out at the 1% level of significance? 1 mark

$p < 0.01 \Rightarrow$ Strong Evidence against H_0.

ie. that $\mu < 200$.

c. The company decided to produce a new type of light globe called Globeplus.

Find an approximate 95% confidence interval for the mean lifetime of the new globes if a random sample of 25 Globeplus globes is tested and the sample mean is found to be 250 weeks. Assume that the standard deviation of the population is 10 weeks. Give your answer correct to two decimal places. 1 mark

$n = 25 \quad \bar{x} = 250 \quad \sigma = 10.$

$\left(250 - 1.96 \times \frac{10}{\sqrt{25}},\ 250 + 1.96 \times \frac{10}{\sqrt{25}}\right).$

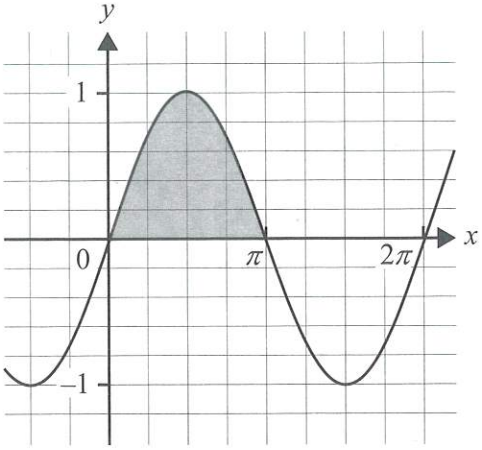
$(250 - 1.96 \times 2,\ 250 + 1.96 \times 2).$

$(250 - 3.92,\ 250 + 3.92)$

$(246.08,\ 253.92).$

Question 4 (4 marks)

a. The shaded region in the diagram below is bounded by the graph of $y = \sin(x)$ and the x-axis between the first two non-negative x-intercepts of the curve, that is, the interval $[0, \pi]$. The shaded region is rotated about the x-axis to form a solid of revolution.



$\cos(2x) = 1 - 2\sin^2(x)$.

$2\sin^2(x) = 1 - \cos(2x)$

$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.

Find the volume, V_s, of the solid formed. 3 marks

$V_s = \pi \int_0^{\pi} \sin^2(x)\, dx$.

$= \frac{\pi}{2} \int_0^{\pi} (1 - \cos(2x))\, dx$.

$= \frac{\pi}{2} \left[x - \sin(2x) \right]_0^{\pi}$

$= \frac{\pi}{2} \left[(\pi - \sin(2\pi)) - [0 - \sin 0) \right]$

$= \frac{\pi}{2} [\pi - 0 - 0 + 0]$

$V_s = \frac{\pi^2}{2}$

b. Now consider the function $y = \sin(kx)$, where k is a positive real constant. The region bounded by the graph of the function and the x-axis between the first two non-negative x-intercepts of the graph is rotated about the x-axis to form a solid of revolution.

Find the volume of this solid in terms of V_s. 1 mark

$\sin(kx) \rightarrow$ Dilation of $\frac{1}{k}$ from y-axis.

Dilated in 1-dimension

$\Rightarrow$ Area Dilated in 1-dimension

$\therefore V = \frac{1}{k} V_s$.

Question 5 (3 marks)

Find the gradient of the curve with equation $e^x e^{2y} + e^{4y^2} = 2e^4$ at the point (2, 1).

$m = \frac{dy}{dx} \rightarrow \frac{d}{dx}\left(e^x e^{2y} + e^{4y^2} = 2e^4\right)$

$e^x e^{2y} + e^x \times 2e^{2y} \times \frac{dy}{dx} + 4e^{4y^2} \times 2y \times \frac{dy}{dx} = 0$

$e^x e^{2y} + \frac{dy}{dx}\left(2e^x e^{2y} + 8ye^{4y^2}\right) = 0$

at (2, 1).

$e^2 \times e^2 + \frac{dy}{dx}\left(2e^2 e^2 + 8 \times 1 \times e^{4\times1^2}\right) = 0$

$e^4 + \frac{dy}{dx}\left(2e^4 + 8e^4\right) = 0$

$\frac{dy}{dx}\left(10e^4\right) = -e^4$

$\frac{dy}{dx} = \frac{-e^4}{10e^4}$

$\frac{dy}{dx} = -\frac{1}{10}$

DO NOT WRITE IN THIS AREA

Question 6 (4 marks)

Consider the three vectors $\underset{\sim}{a} = -\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - 8\underset{\sim}{j} + 5\underset{\sim}{k}$ and $\underset{\sim}{c} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + \left|1 - p^2\right|\underset{\sim}{k}$, where p is a real constant.

Find the values of p for which the three vectors are $\boxed{\text{linearly independent.}}$

Linearly Dependent when $\underset{\sim}{c} = m\underset{\sim}{a} + n\underset{\sim}{b}$

$3\underset{\sim}{i} + 2\underset{\sim}{j} + |1-p^2|\underset{\sim}{k} = m(-\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k}) + n(2\underset{\sim}{i} - 8\underset{\sim}{j} + 5\underset{\sim}{k})$.

$\underset{\sim}{i}$ component: $3 = -m + 2n$. $\quad\times 4$

$\underset{\sim}{j}$ component: $2 = 6m - 8n$

$\underset{\sim}{k}$ component: $|1-p^2| = -3m + 5n$.

$m = 7 \quad n = 5$

$12 = -4m + 8n$

$2 = 6m - 8n$ (+)

$14 = 2m$

$m = 7$.

$3 = -7 + 2n$

$10 = 2n$

$n = 5$.

$|1-p^2| = -3\times7 + 5\times5$

$|1-p^2| = -21 + 25$

$|1-p^2| = 4$

$1 - p^2 = \pm 4$

$p^2 = 1 \pm 4$

$p \in R$.

$p^2 = 1 + 4 = 5$

$p = \pm\sqrt{5}$.

This is for Linear Dependence.

For Linear Independence.

$p \in R \setminus \{\pm\sqrt{5}\}$

Note: Use strategy of finding when Linearly dependent.

We want Linear Independent

Thus we want everything else.

Question 7 (5 marks)

The velocity of a particle satisfies the differential equation $\frac{dx}{dt} = x\sin(t)$, where x centimetres is its displacement relative to a fixed point O at time t seconds.

Initially, the displacement of the particle is 1 cm.

a. Find an expression for x in terms of t. 3 marks

$\int \frac{1}{x}\, dx = \int \sin(t)\, dt.$

$\log_e|x| = -\cos(t) + c.$

$t = 0 \quad x = 1$

$\log_e(1) = -\cos(0) + c$

$0 = -1 + c$

$c = 1$

$\log_e|x| = 1 - \cos(t)$

$|x| = e^{1-\cos(t)}. \qquad x = e^{1-\cos(t)}$

b. Find the maximum displacement of the particle and the times at which this occurs. 2 marks

$\frac{dx}{dt} = \frac{d}{dt}\left(e^{1-\cos(t)}\right) = 0$

Let $u = 1 - \cos(t)$. $\quad x = e^{u}$

$\frac{du}{dt} = \sin(t). \quad \frac{dx}{du} = e^{u}$

$\frac{dx}{dt} = \frac{dx}{du} \times \frac{du}{dx}$

$= e^{u} \times \sin(t).$

$0 = e^{1-\cos(t)} \times \sin(t).$

$e^{1-\cos(t)} = 0$ OR $\sin(t) = 0$

No Sol^{n}

$t = 0, \pi, 2\pi, 3\pi, \ldots$

$x = e^{1-\cos(0)}, e^{1-\cos(\pi)}, e^{1-\cos(2\pi)}, e^{1-\cos(3\pi)}, \ldots$

$= e^{0}, e^{2}, e^{0}, e^{2}, \ldots$

$= 1, e^{2}, 1, e^{2}, \ldots$

Max Displacement $= e^{2}$ at $t = \pi, 3\pi, 5\pi, \ldots$

DO NOT WRITE IN THIS AREA

Question 8 (4 marks)

a. Solve $z^2 + 2z + 2 = 0$ for z, where $z \in C$. 1 mark

Quadratic Formula.

$z = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times 2}}{2 \times 1}$

$= \frac{-2 \pm \sqrt{4-8}}{2}$

$= \frac{-2 \pm \sqrt{-4}}{2}$

$= \frac{-2 \pm \sqrt{4i^2}}{2}$

$z = \frac{-2 \pm 2i}{2}$

$= \frac{2(-1 \pm i)}{2}$

$= -1 \pm i$

b. Solve $z^2 + 2\bar{z} + 2 = 0$ for z, where $z \in C$. 3 marks

Let $z = a + bi$ $\quad \bar{z} = a - bi$

$(a+bi)^2 + 2(a - bi) + 2 = 0$

$a^2 + 2abi + b^2i^2 + 2a - 2bi + 2 = 0$

$a^2 - b^2 + 2a + 2 + 2b(a-1)i = 0$

Equate Real and Imaginary Components

$a^2 - b^2 + 2a + 2 = 0$

$2b(a-1) = 0$.

$2b = 0$ OR $a - 1 = 0$

$b = 0$ $\quad a = 1$.

When $b = 0$

$a^2 + 2a + 2 = 0$.

No Real Sol^n.

$\Delta = 2^2 - 4 \times 1 \times 2$

$\Delta = -4$

When $a = 1$

$1 - b^2 + 2 + 2 = 0$

$b^2 = 5$

$b = \pm\sqrt{5}$

$z = 1 \pm \sqrt{5}\, i$

DO NOT WRITE IN THIS AREA

Question 9 (8 marks)

Let $\underset{\sim}{r}(t) = (-1 + 4\cos(t))\underset{\sim}{i} + \frac{2}{\sqrt{3}}\sin(t)\underset{\sim}{j}$ and $\underset{\sim}{s}(t) = (3\sec(t) - 1)\underset{\sim}{i} + \tan(t)\underset{\sim}{j}$ be the position vectors relative to a fixed point O of particle A and particle B respectively for $0 \le t \le c$, where c is a positive real constant.

a. **i.** Show that the cartesian equation of the path of particle A is $\frac{(x+1)^2}{16} + \frac{3y^2}{4} = 1$. 1 mark

$x = -1 + 4\cos(t)$. $y = \frac{2}{\sqrt{3}}\sin(t)$

$\cos(t) = \frac{x+1}{4}$ $\sin(t) = \frac{\sqrt{3}y}{2}$.

$\sin^2(t) + \cos^2(t) = 1$.

$\left(\frac{\sqrt{3}y}{2}\right)^2 + \left(\frac{x+1}{4}\right)^2 = 1$

$\frac{3y^2}{4} + \frac{(x+1)^2}{16} = 1$.

ii. Show that the cartesian equation of the path of particle A in the first quadrant can be written as $y = \frac{\sqrt{3}}{6}\sqrt{-x^2 - 2x + 15}$. 1 mark

$\frac{3y^2}{4} = 1 - \frac{(x+1)^2}{16}$.

$\frac{3y^2}{4} = \frac{16 - (x+1)^2}{16}$.

$3y^2 = \cancel{4} \times \frac{16 - (x+1)^2}{\cancel{16}\,4}$

$3y^2 = \frac{16 - (x^2 + 2x + 1)}{4}$

$y^2 = \frac{15 - x^2 - 2x}{12}$.

$\sqrt{12} = 2\sqrt{3}$

$y = \frac{\sqrt{-x^2 - 2x + 15}}{\sqrt{12}}$

$y = \frac{\sqrt{-x^2 - 2x + 15}}{2\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}}$

$y = \frac{\sqrt{3}}{6}\sqrt{-x^2 - 2x + 15}$

b. **i.** Show that the particles A and B will collide. 1 mark

$\underset{\sim}{r}(t) = \underset{\sim}{s}(t)$

$-1 + 4\cos(t) = 3\sec(t) - 1$

$4\cos(t) = \frac{3}{\cos(t)}$

$4\cos(t) - \frac{3}{\cos(t)} = 0$

$\frac{4\cos^2(t) - 3}{\cos(t)} = 0$.

$4\cos^2(t) - 3 = 0$

$\cos^2(t) = \frac{3}{4}$

$\cos(t) = \pm\frac{\sqrt{3}}{2}$

$\frac{2}{\sqrt{3}}\sin(t) = \tan(t)$

$\frac{2}{\sqrt{3}}\sin(t) - \tan(t) = 0$

$\frac{2\sin t}{\sqrt{3}} - \frac{\sin(t)}{\cos(t)} = 0$

$\frac{2\sin t\cos t - \sqrt{3}\sin(t)}{\sqrt{3}\cos(t)} = 0$

$\sin(t)(2\cos t - \sqrt{3}) = 0$

$\sin t = 0$ $\cos(t) = \frac{\sqrt{3}}{2}$

$\cos(t) = \frac{\sqrt{3}}{2}$ common to both.

Thus will collide when $\cos(t) = \frac{\sqrt{3}}{2}$.

$t = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$.

ii. Hence, find the coordinates of the point of collision of the two particles. 1 mark

$\cos(t) = \frac{\sqrt{3}}{2} \rightarrow \sin(t) = \frac{1}{2}$. Note: $t = \frac{\pi}{6}$.

$\underset{\sim}{r}\left(\frac{\pi}{6}\right) = \underset{\sim}{s}\left(\frac{\pi}{6}\right) = \left(-1 + \cancel{4}^{2} \times \frac{\sqrt{3}}{\cancel{2}}\right)i + \frac{\cancel{2}}{\sqrt{3}} \times \frac{1}{\cancel{2}} j$

Point of Intersection $\left(-1 + 2\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

Question 9 – continued

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c. **i.** Show that $\frac{d}{dx}\left(8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{-x^2-2x+15}}{2}\right)=\sqrt{-x^2-2x+15}$. 2 marks

$= 8 \times \frac{1}{\sqrt{1-(\frac{x+1}{4})^2}} \times \frac{1}{4} + \frac{1}{2} \times \sqrt{-x^2-2x+15} + \frac{(x+1)}{2} \times \frac{1}{2}(-x^2-2x+15)^{-\frac{1}{2}} \times (-2x-2)$

$= \frac{2}{\sqrt{1-\frac{(x+1)^2}{16}}} + \frac{\sqrt{-x^2-2x+15}}{2} + \frac{(x+1)(-2x-2)}{4\sqrt{-x^2-2x+15}}$

$= \frac{2}{\sqrt{\frac{16-(x+1)^2}{16}}} + \frac{\sqrt{-x^2-2x+15}}{2} + \frac{-8(x+1)^2}{2\cancel{4}\sqrt{-x^2-2x+15}}$

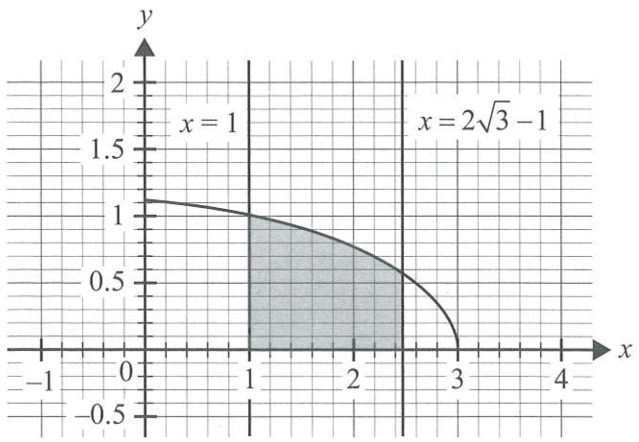
$= \frac{8}{\sqrt{16-(x+1)^2}} + \frac{(1-(x+1)^2)-(x+1)^2}{2\sqrt{1-(x+1)^2}}$

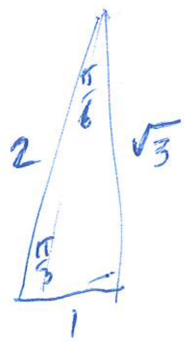
$= \frac{8}{\sqrt{16-(x+1)^2}} + \frac{8-(x+1)^2}{\sqrt{1-(x+1)^2}}$

$= \frac{16-(x+1)^2}{\sqrt{16-(x+1)^2}}$

$= \sqrt{16-(x+1)^2} = \sqrt{-x^2-2x+15}$

ii.





Hence, find the area bounded by the graph of $y=\frac{\sqrt{3}}{6}\sqrt{-x^2-2x+15}$, the x-axis and the lines $x = 1$ and $x=2\sqrt{3}-1$, as shown in the diagram above. Give your answer in the form $\frac{a\sqrt{3}\pi}{b}$, where a and b are positive integers. 2 marks

$\int_1^{2\sqrt3-1} \frac{\sqrt3}{6}\sqrt{-x^2-2x+15}\,dx$

$= \frac{\sqrt3}{6}\int_1^{2\sqrt3-1}\sqrt{16-(x+1)^2}\,dx$

$= \frac{\sqrt3}{6}\left[8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{16-(x+1)^2}}{2}\right]_1^{2\sqrt3-1}$

$= \frac{\sqrt3}{6}\left[\left(8\arcsin\left(\frac{2\sqrt3}{4}\right)+\frac{(2\sqrt3)\sqrt{16-(2\sqrt3)^2}}{2}\right)-\left(8\arcsin\left(\frac{2}{4}\right)+\frac{2\sqrt{16-(2)^2}}{2}\right)\right]$

$= \frac{\sqrt3}{6}\left[\left(8\times\frac{\pi}{3}+\cancel{\frac{2\sqrt3\times4}{2}}\right)-\left(8\times\frac{\pi}{6}+\cancel{\frac{2\times\sqrt{12}}{2}}\right)\right]$

$= \frac{\sqrt3}{6}\left[\frac{8\pi}{3}-\frac{8\pi}{6}\right] = \frac{\sqrt3}{\cancel{6}3}\left[\frac{\cancel{8}4\pi}{\cancel{6}3}\right] = \frac{2\sqrt3\pi}{9}$

END OF QUESTION AND ANSWER BOOK

9. c) i - Detail

$$\frac{d}{dx}\left(8\arcsin\left(\frac{x+1}{4}\right)+\frac{(x+1)\sqrt{-x^2-2x+15}}{2}\right)$$

Chain Rule

$$\overset{2}{\cancel{8}}\times\frac{1}{\sqrt{1-\left(\frac{x+1}{4}\right)^2}}\times\frac{1}{\cancel{4}_1}$$

$$\frac{2}{\sqrt{1-\frac{(x+1)^2}{16}}}$$

$$\frac{2}{\sqrt{\frac{16-(x+1)^2}{16}}}$$

$$\frac{2}{\frac{\sqrt{16-(x+1)^2}}{4}}$$

$$2\times\frac{4}{\sqrt{16-(x+1)^2}}$$

$$\frac{8}{\sqrt{16-(x+1)^2}}$$

Product Rule & Chain on $(-x^2-2x+15)^{\frac{1}{2}}$

$$1\times\frac{\sqrt{-x^2-2x+15}}{2}+\frac{(x+1)}{2}\times\frac{1}{2}(-x^2-2x+15)^{-\frac{1}{2}}\times(-2x-2)$$

$$\frac{\sqrt{-x^2-2x+15}}{2}+\frac{(x+1)(-2x-2)}{4\sqrt{-x^2-2x+15}}$$

$$\frac{\sqrt{-x^2-2x+15}}{2}-\frac{\cancel{2}^{1}(x+1)^2}{{}_2\cancel{4}\sqrt{-x^2-2x+15}}$$

$$\frac{(-x^2-2x+15)-(x+1)^2}{2\sqrt{-x^2-2x+15}}$$

from a ii $-x^2-2x+15=16-(x+1)^2$.

$$\frac{16-(x+1)^2-(x+1)^2}{2\sqrt{16-(x+1)^2}}$$

$$\frac{\cancel{2}(8-(x+1)^2)}{\cancel{2}\sqrt{16-(x+1)^2}}$$

$$\frac{8-(x+1)^2}{\sqrt{16-(x+1)^2}}$$

$$=\frac{8}{\sqrt{16-(x+1)^2}}+\frac{8-(x+1)^2}{\sqrt{16-(x+1)^2}}$$

$$=\frac{16-(x+1)^2}{\sqrt{16-(x+1)^2}}$$

$$=\sqrt{16-(x+1)^2}$$

$$=\sqrt{-x^2-2x+15}$$