M.8.2.5 *Length Contraction*

Videos: Hewitt Drewit – Length Contraction

Relativity paradox – Length Contraction

Einstein proposed that **time** was **not absolute** and that it depended on the frame of reference in which it was measured. He then went further and suggested that the same also applied for **distances** and **lengths** in the direction of the motion.

Let us imagine that Adam's train carriage has now headed off at speed *v* towards a distant star. If the star is L_0 from the train when measured from Eve's stationary (relative to the star) frame of reference.

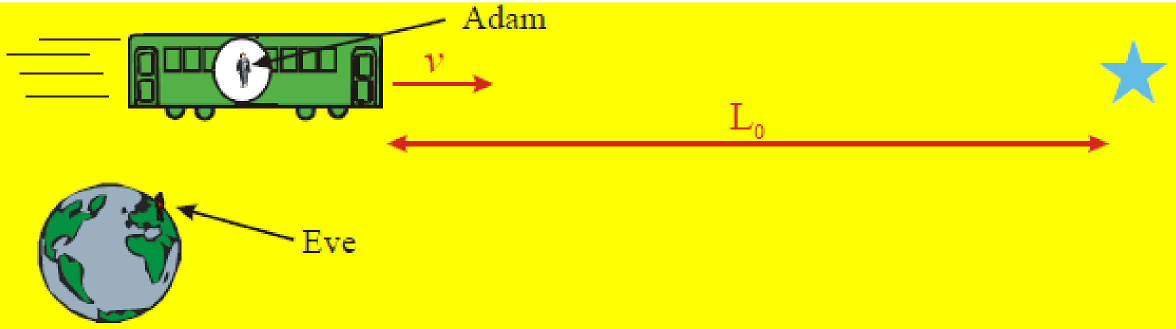

Figure 7.13 From Eve's frame of reference, the distance between the train and the star is L_0.

If the time taken to reach the star is different in each frame of reference, less time for Adam. Then, the distance L relative to his frame of reference is less than L_0 as measured by Eve.

Einstein's **length contraction** equation incorporates the Lorentz factor. This equation shows that an object with a **proper length** of L_0, when measured at rest, will have a shorter length L, parallel to the motion of its moving frame of reference when measured by an observer that is in a stationary frame of reference.

$$L = \frac{L_0}{\gamma}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

L_0 is the proper length i.e. the length measured at rest, in the stationary frame of reference.

L is the length in the moving frame, measured by an observer.

Einstein said that when viewed from a stationary frame, the distances travelled *in the direction of the motion* for the moving frame would be decreased by a factor of γ. There was **no contraction perpendicular to the motion**. Einstein went further and said that it was not just the distances and lengths in the direction of motion; it was **space itself** that was **contracting**.

Example

Assume *Gedanken* conditions exist in this example. A stationary observer on Earth sees a very fast car travelling by at 2.50×10^8 m s^{-1}. When stationary, the car is 3.00 m long. Calculate the length of the car as seen by the stationary observer. Use $c = 3.00 \times 10^8$ m s^{-1}.

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1.809$ $\qquad L = \frac{3.00}{1.809} = 1.658$

Example

Assume *Gedanken* conditions exist in this example. A pilot of a spaceship travelling at $0.997c$ is travelling from Earth to the Moon. The proper distance from the Earth to the Moon is 384 400 km. When the pilot looks out of the window, the distance between the Earth and the Moon looks much less than that. Calculate the distance that the pilot sees.

$\gamma = \frac{1}{\sqrt{1-\frac{(0.997\,c)^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.994\,c^2}{c^2}}} = \frac{1}{\sqrt{1-0.994}} = 12.9$ $\qquad L = \frac{384\,400}{12.9} = 29\,798$

NOTE:

PROPER TIME AND PROPER LENGTH

The time t_0 and the length L_0 are referred to as the proper time and proper length.

They are the quantities measured by the observer, who is in the same frame of reference as the event or the object being measured.

Text Questions: Page 224 All Questions