

M.8.2.7 Relativistic Mass

Video: Mass-Energy Equivalence or $E = mc^2$

Einstein worked out that as the **speed** of a particle increases, so too does its **mass**. This **mass increase** is not an effect that you will notice as you travel on the school bus, but it does become apparent at very high speeds. Mass, along with time and distance, cannot be considered to be an absolute quantity. Its value depends on the speed of its frame of reference.

- The mass of a particle as measured in its own frame of reference is called the **rest mass, m_0** .
- The mass of a particle as measured from another frame of reference is called the **relativistic mass, m**

Relativistic mass $m = m_0 \times \gamma$

This helps to explain why it is not possible to accelerate particles such as electrons so that they are travelling at the speed of light. As the electron goes faster, its mass increases. This makes it more difficult to make it go faster, so a larger force is required to make it accelerate. As its speed approaches $3.0 \times 10^8 \text{ m s}^{-1}$, the mass of the electron approaches infinity, so it can never be made to travel at the speed of light. Einstein's Special Theory of Relativity can precisely predict and explain the behaviour of these particles.

According to the postulates of special relativity, certain properties are dependent on the reference frame in which they are observed.

Example 50 (2005 Q1, 3 marks)

In column 2 of the table below, indicate whether the entry in column 1 is always the same (S), or may sometimes be different (D).

column 1	column 2
The mass of an electron measured at rest	
The time interval between two given events	
The distance between two given events	

Example 51 (2006 Q10, 2 marks)

Which of the following (**A - D**) is true with regard to the electron's speed, and its mass, as the speed of the electron approaches c ?

- A. Its speed increases slightly while its mass remains fixed at $9.11 \times 10^{-31} \text{ kg}$.
 - B. Its speed increases slightly while its mass increases substantially.
 - C. Its speed increases substantially while its mass increases slightly.
 - D. Both its speed and its mass continue to increase at a steady rate.
-

M.8.2.8 Relativistic Momentum

If a rocket ship is travelling at $0.99c$, why can't it simply turn on its rocket motor and accelerate up to c , or more? A full answer to this question was not given in Einstein's original 1905 paper on relativity. Some years later he showed that as the speed of a spaceship approaches c , its momentum increases, but this is not reflected in a corresponding increase in speed.

To Newton, infinite momentum would mean infinite mass or infinite speed. But not so in relativity. Einstein showed that a new definition of momentum was required. It is

$$p = \gamma mv$$

where γ is the Lorentz factor (γ is always ≥ 1).

At higher speeds γ grows dramatically, and so does relativistic momentum. As speed approaches c , γ approached infinity! No matter how close to c an object is pushed, it will still require infinite impulse to give it the last bit of speed needed to reach c – clearly impossible.

M.8.2.9 Einstein's Famous Equation

Einstein linked not only space and time but also mass and energy. A piece of matter, even at rest and not interacting with anything else, has an “energy of being”. This is called its *rest energy*. Einstein concluded that it takes energy to make mass and that energy is released if mass disappears.

This linkage has resulted in the term mass-energy.

The **mass-energy** of any object is given by $E = mc^2$.

With mass-energy, a moving body has kinetic energy and rest energy.

$$\begin{aligned}\therefore E &= E_k + E_{\text{rest}} \\ \therefore mc^2 &= E_k + m_0c^2 \\ \therefore E_k &= mc^2 - m_0c^2\end{aligned}$$

Substituting $m = m_0\gamma$

$$\begin{aligned}\therefore E_k &= m_0\gamma c^2 - m_0c^2 \\ \therefore E_k &= (\gamma - 1)m_0c^2\end{aligned}$$

When particles are travelling at high speeds (particularly greater than 10% of c) that the KE is given by

$$E_k = (\gamma - 1)m_0c^2, \text{ where } m_0 = \text{rest mass.}$$

An electron in a colour television tube moves so fast that its 'mass energy' is 21% greater than the mass-energy of an electron at rest. The rest mass-energy of an electron is 8.2×10^{14} J.

Example 52 (2004 Sample Q3, 2 marks)

What is the kinetic energy of the electron?

Example 53 (2004 Sample Q4, 2 marks)

What is the speed of the moving electron?

Example 54 (2004 Sample Q6, 2 marks)

An object has a rest energy of 18 J. Explain what this means.