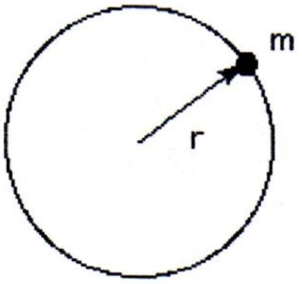
How Fast Can Things Go?

Reference: Heinemann Physics 12 4th Edition Chapters 5 – 7 Pages 147 – 269

M.4 Circular Motion

M.4.1 Uniform Circular Motion

Let us consider an object moving in a circle with a constant speed.



The speed of the object will be given by:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{\text{circumference}}{\text{period}}$$

$$v = \frac{2\pi r}{T}$$

The period (T) is the time for one revolution.

Example:
An athlete is swinging a hammer of mass 7.0 kg in a circular path of radius 1.5 m. Calculate the speed of the hammer if it completes 3.0 revolutions per second.

Solution

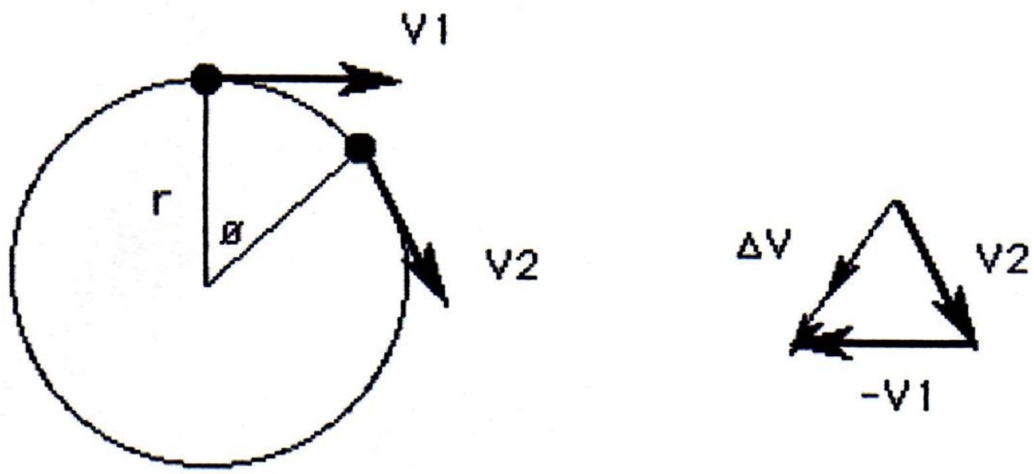
The period of the hammer is: $T = \frac{1}{f} = \frac{1}{3.0} = 0.33s$

The speed is: $v = \frac{2\pi r}{T} = \frac{2\times\pi\times1.5}{0.3333} = 28\,ms^{-1}$

M.4.2 Centripetal Acceleration

Since the direction of travel is continually changing the velocity of this object will also be constantly changing. Because the velocity is changing there must be an acceleration.

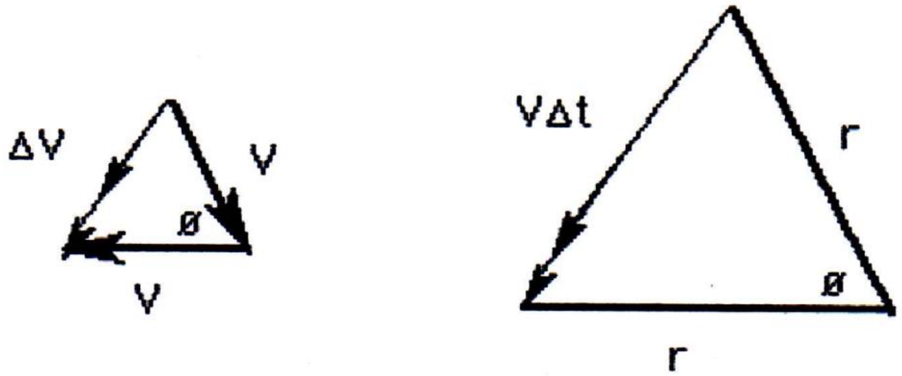
To find the acceleration we must first find the change in velocity.



$\Delta V = V_2 - V_1$

acceleration $= \dfrac{\Delta v}{\Delta t}$

If we take a very small time interval then the distance travelled will approximate a straight line and will be given by vΔt. We will have two triangles as shown below:



and from the similar triangles we get:

$$\frac{\Delta V}{V} = \frac{V\Delta t}{r}$$

$$\frac{\Delta V}{\Delta t} = \frac{V^2}{r}$$

Hence the magnitude of the acceleration is

$$a = \frac{v^2}{r}$$

using $v = \frac{2\pi r}{T}$

we get:

$$a = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = \frac{4\pi^2 r}{T^2}$$

using $T = \frac{1}{f}$

we get $a = 4\pi^2 r f^2$

The direction of the acceleration is towards the center of the circle.

The net force will also be towards the centre of the circle, it is known as the centripetal force and is given by:

$$F = ma = \frac{m\,4\pi^2 r}{T^2}$$

$$F = \frac{mv^2}{r}$$

Example:

An athlete in a hammer throw event is swinging the hammer of mass 7.0 kg in a horizontal circular path. Calculate the tension in the wire if the hammer is moving at 20 m s^{-1} in a circle of radius 1.6 m

Solution

The centripetal acceleration is:

$$a = \frac{v^2}{r} = \frac{20^2}{1.6}$$

$= 250$ m s^{-2} towards the centre

The tension is producing circular motion:

$\boldsymbol{F}t = \Sigma \boldsymbol{F}$

$= m\boldsymbol{a}$

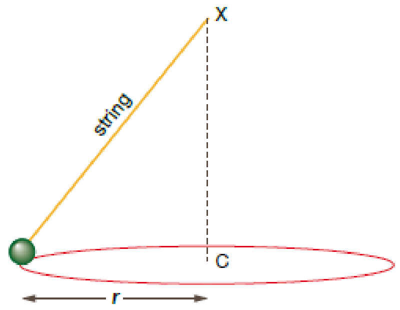
$= 7.0 \times 250$

$= 1.8 \times 103$ N towards the centre

M.4.3 Applications of Circular Motion

M.4.3.1 Ball on a String

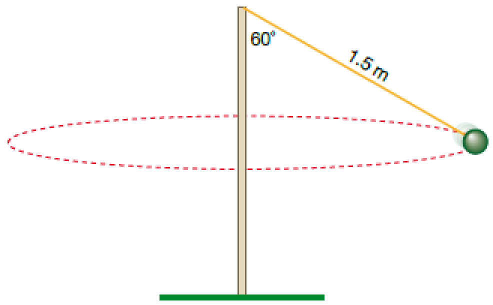
Consider a ball on a string which is rotating in a horizontal circle whose radius is less than the length of the string, as shown in the diagram below.



This arrangement is known as a conical pendulum. The faster the ball rotates the larger the angle between the string and the vertical. When this angle is known trigonometry and the circular motion formulae can be used to find a solution.

Example.

During a game of Totem Tennis, the ball of mass 150 g is swinging freely in a horizontal circular path. The cord is 1.50 m long and is at an angle of 60.0° to the vertical shown in the diagram.



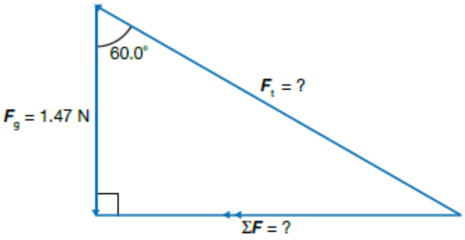
a) Calculate the radius of the ball's circular path.
b) Draw and identify the forces that are acting on the ball at the instant shown in the diagram.
c) Determine the net force that is acting on the ball at this time.
d) Calculate the size of the tensile force in the cord.
e) How fast is the ball travelling at this time?

Sol^n
a) Trigonometry and a distance triangle can be used to work this out.

$r = 1.50 \times \sin 60.0° = 1.30$ m

b) There are two forces acting — the tension in the cord, $\boldsymbol{F}_t$, and gravity, $\boldsymbol{F}_g$.

c)



ΣF = 1.47 × tan60.0° = 2.55 N towards the left

d) $F_t = \frac{1.47}{\cos 60°}$ = 2.94 N along direction of the cord

e) ΣF = 2.55 N, m = 0.150 kg, r = 1.30 m, v = ?

$$\Sigma F = \frac{m v^2}{r}$$

$$2.55 = \frac{0.150\, v^2}{1.30}$$

$$\Rightarrow v = 4.70 \text{ m s}^{-1}$$

Text Questions: Page 164 All Questions