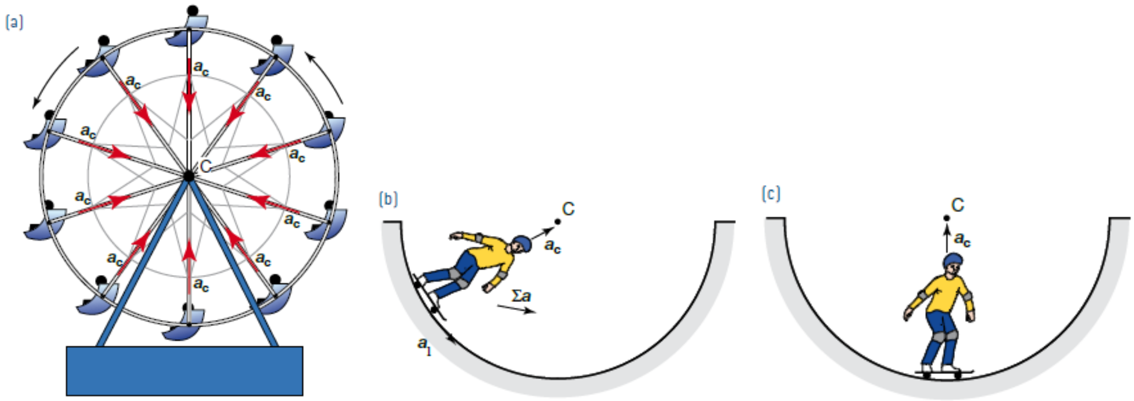
## *M.4.4 Circular Motion in the Vertical Direction*

There are many situations where circular motion occurs in the vertical direction, fair ground rides, skater in a half pipe and cars going over humps.

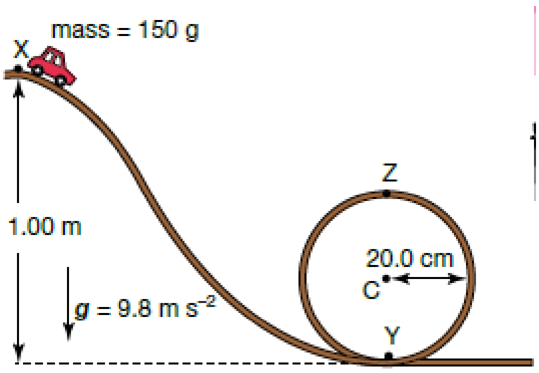


In these cases the speed is generally not uniform. The speed will be greater at bottom the of the circle than the top.
The force acting on the object at any time will be given by:

$$\Sigma F = F_c + mg$$

When solving problems involving circular motion the use of conservation of energy along with the circular motion formulae can be very useful.

Example
A student arranges a toy car track with a vertical loop of radius 0.200 m, as shown.




A car of mass 150 g is released from a height of 1.00 m at point X. The car rolls down the track and travels around the loop. Assuming $g$ is 9.80 m $s^{-2}$, and ignoring friction, calculate:
**a** the speed of the car as it reaches the bottom of the loop, point Y
**b** the centripetal acceleration of the car at point Y
**c** the normal reaction force from the track at point Y
**d** the speed of the car as it reaches the top of the loop at point Z
**e** the apparent weight of the car at point Z
**f** the release height from which the car will just maintain contact with the track as it travels past point Z.

Sol$^n$

a) At X the car has gravitational potential energy, at Y this $U_g$ has been transformed into Kinetic energy. Thus $U_g$ at X = $E_k$ at Y

$$m\,g\,h = \frac{1}{2}\,m\,v^2$$

$$0.150 \times 9.8 \times 1.00 = \frac{1}{2} \times 0.150 \times v^2$$

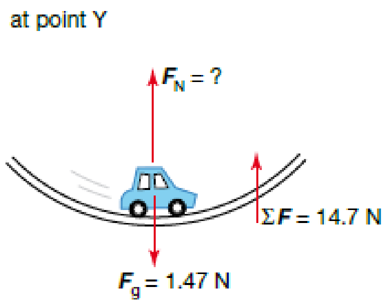
$$v = \sqrt{19.6} = 4.43 \text{ m s}^{-1}$$

b) $\vec{a} = \frac{v^2}{r}$

$$\vec{a} = \frac{19.6}{0.200}$$

$\vec{a} = 98.1 \text{ m s}^{-2}$ towards C, i.e. upwards

c)




$\Sigma \boldsymbol{F} = \boldsymbol{F}_N + \boldsymbol{F}_g$

but $\Sigma \boldsymbol{F} = m\,\boldsymbol{a}$

$0.150 \times 98.1 = \mathbf{F}_N + 0.150 \times -9.80$ ( the –ve indicates direction)

$14.7 = \boldsymbol{F}_N - 1.47$

$\therefore \boldsymbol{F}_N = 14.7 - -1.47 = 16.2$ N upwards

d) Some $E_k$ will be transformed into $U_g$

$\Sigma E = E_k + U_g$

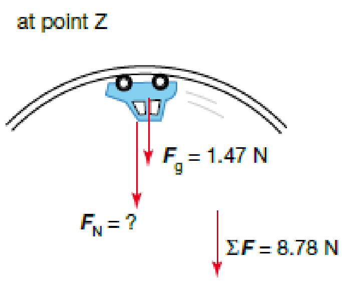
$\therefore 1.47 = \frac{1}{2}mv^2 + 0.150 \times 9.80 \times 0.400$

$\therefore 1.47 = \frac{1}{2}mv^2 + 0.588$

$\therefore 0.88 = 0.5 \times 0.150 \times v^2$

$\therefore v = \sqrt{11.7} = 3.42 \text{ m s}^{-1}$

e)




$\Sigma \boldsymbol{F} = \boldsymbol{F}_N + \boldsymbol{F}_g$

$$m\,\frac{v^2}{r} = \boldsymbol{F}_N + m\,g$$

$$0.150 \times -\frac{11.7}{0.200} = \boldsymbol{F}_N + 0.150 \times -9.80$$

$-8.78 = \boldsymbol{F}_N - 1.47$

$\therefore$ $\boldsymbol{F}_N$ = 7.31 N down. This is about five times greater than the actual weight.

f) At the point where the car just starts to lose contact with the track the normal reaction force is zero, thus $\Sigma\mathbf{F} = \mathbf{F}_g$

$\Sigma\boldsymbol{F} = \boldsymbol{F}_g$

$m\,\frac{v^2}{r} = m\,g$

$v = \sqrt{g\,r}$

$v = \sqrt{9.80 \times 0.200} = 1.40$ m s$^{-1}$

The total energy of the toy car at point Z is therefore:

$\Sigma E = E_k + U_g$

$= (0.5 \times 0.150 \times 1.40^2) + (0.150 \times 9.80 \times 0.400)$

$= 0.147 + 0.588$

$= 0.735$ J

So when the car is released, its height needs to be such that it has 0.735 J of gravitational potential energy:

$\therefore mgh = 0.735$

$\therefore 0.150 \times 9.80 \times h = 0.735$, so $h = 0.500$ m

The student should release the car from a height of 50.0 cm for it to complete the loop.

**Text Questions:** Page 177, 178 All Questions