Physics with Synno – Motion-2 – Lesson 20

M.7 Work and Energy

M.7.1 Work

Prac #7: EXPT 2.7 Work Done along an Inclined Plane

From the ideas we have about work we can say that work requires some effort. E.g. pushing a wheel barrow. Thus some force is being applied over some distance.
From common sense we get

Work = Force × Displacement

or $W = \mathbf{F} \times x$

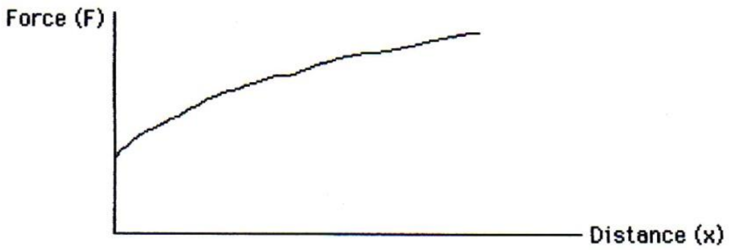
The units for work are the Joule (J)

Example:
Find the work done against a frictional force of 7 N. If the fridge is kept at a constant velocity of 2 m s^{-1} for a distance of 10 m.

Constant velocity → zero acceleration → applied force = 7N
$W = \mathbf{F} \times x$
$W = 7 \times 10$
W = 70 J

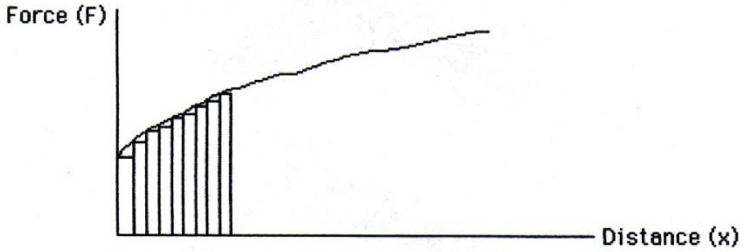
M.7.2 Work From Graphs

Suppose we have a force-distance graph that looks like this



How do we calculate the work done?

Firstly we replace the curve by a series of steps of constant force.



Then what we have is a series of small rectangles, with an area of $f \times x$.

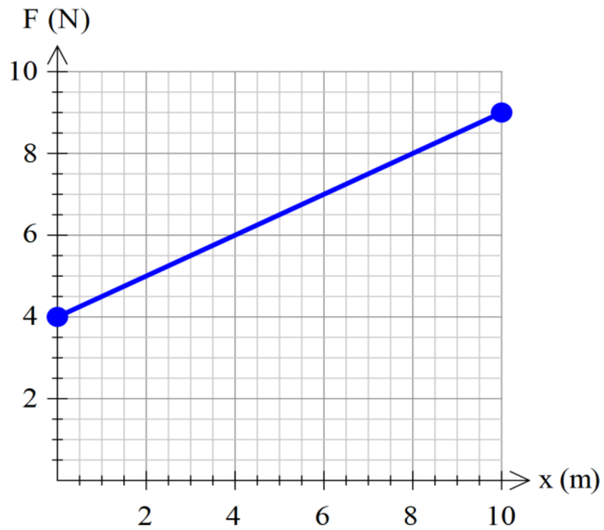
So if we add up all the rectangles.
We get, $\Sigma(F \times x)$ or $\int F(x)\,dx$ which is the area under the graph.

So **Work done = area under *F-x* graph**

This follows for all *F-x* graphs even those where F is not constant.

Examples: Find the work done in the following cases

1.



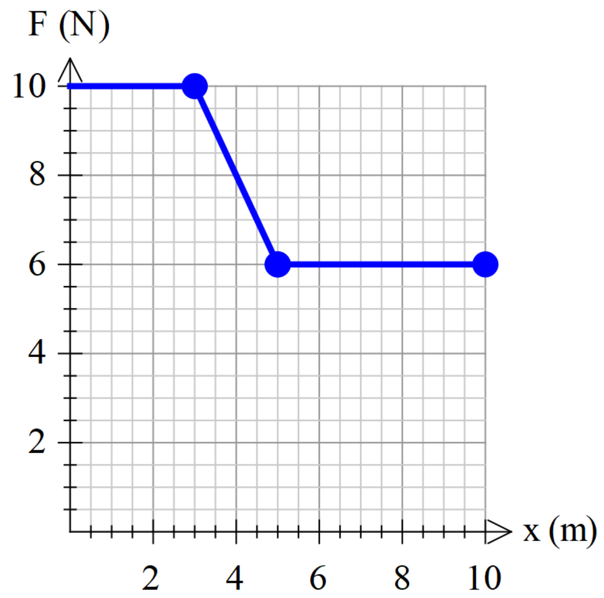
Work = Area
There are a number of ways to calculate the area.

Rectangle + Triangle

$10 \times 4 + \frac{1}{2} \times 10 \times 5 = 65$

Work = 65 J

2.



Work = Area
There are a number of ways to calculate the area

Rectangle + triangle + rectangle

$(10 \times 3) + \left(\frac{1}{2} \times 2 \times 4\right) + (7 \times 6) = 76$

Work = 76 J

M.7.3 The Scalar Nature of Work

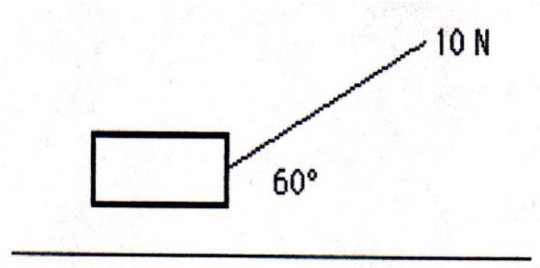
Work is a scalar but force and displacement are vectors. What happens if the force applied and the displacement are not in the same direction? We then need to take the component of the force in the direction of the displacement.

So Work = component of force in direction of displacement ×distance

Work = F *x* cosθ

Where θ is the angle between the applied force and the direction of motion.

Example:
Find the work done by the force if the distance moved is 6 m.



Work = F *x* cosθ
Work = 10 × 6 × cos 60°
Work = 30 J

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M.7.4 Energy

When we do work, such as pushing a wheel barrow, we get tired or use up the quantity known as energy. ***ENERGY*** is the ability to do work.

Energy does not disappear, but is either

1) transferred to another object

or 2) transformed into another kind.

Thus we formulate the principle of conservation of energy which states that

Energy is neither created or destroyed

we say that

Work done by an object = transfer of energy from that object
and Work done on an object = gain in energy to that object

or $\mathbf{W = \Delta E}$

The units for energy are the same as the units for work, the Joule (J).

Example:
If a man does 200 J of work pushing a wheel barrow, he transfers 200 J of energy to the wheel barrow.

M.7.5 Types of Energy

M.6.7.1 Kinetic Energy

We define kinetic energy as the energy a body has when it is in motion. We can derive an expression for kinetic energy.

Consider an object of mass, m, originally at rest being acted upon by a force of F N for a distance of d m. No friction.

We have

Work done = energy gain = final K.E. (in this case)

$\therefore$ Final K.E. $= F \times x$
$= m\,a \times x$ (eq^n 1)

Evaluating the acc^n using constant acc^n formula

$$v^2 = u^2 + 2ax$$

u = 0 v = v x = d a = a

$$v^2 = 0 + 2ad$$

$$v^2 = 2ad$$

$$\Rightarrow \quad a = \frac{v^2}{2d}$$

Now substitute into eq^n 1

$$\text{Final K.E.} = \frac{m\ v^2}{2x} \times x$$

$$= \tfrac{1}{2}\, m\, v^2$$

so **E_K =** $\tfrac{1}{2}\, m\, v^2$

In fact work done = change in kinetic energy = Final K.E. – Initial K.E.

$$W = \Delta E_k = \tfrac{1}{2}\, m\, v^2 - \tfrac{1}{2}\, m\, u^2$$

Examples:

1. A body of mass 6 Kg has a speed of 3 m s^{-1}. What is its K.E.?

   E_K = $\tfrac{1}{2}\, m\, v^2$
   E_K = $\tfrac{1}{2} \times 6 \times 3^2$
   E_K = 27 J

2. A body of mass 4 Kg with a speed of 3 m s^{-1} accelerates to a speed of 6 m s^{-1}. What is
   a) the change in K.E.

   $\Delta E_k = \tfrac{1}{2}\, m\, v^2 - \tfrac{1}{2}\, m\, u^2$
   $\Delta E_k = \tfrac{1}{2} \times 4 \times 6^2 - \tfrac{1}{2} \times 4 \times 3^2$
   $\Delta E_k = 72 - 18$
   $\Delta E_k = 54$ J

   b) the work done on the body

   $W = \Delta E$
   W = 54 J

Video: Physics of Car Crashes #227

M.7.5.2 Potential Energy

The potential energy is the energy stored within a body. The symbol used to represent potential energy is U. Usually followed by a subscript indicating what type.

M.7.5.2.1 Elastic Potential Energy

Prac #8: Expt 2.8 Hooke's Law

Springs can store energy when they are stretched or compressed. We can store the energy in the spring by applying a force to alter its length, thus we are doing work on the spring.
we have

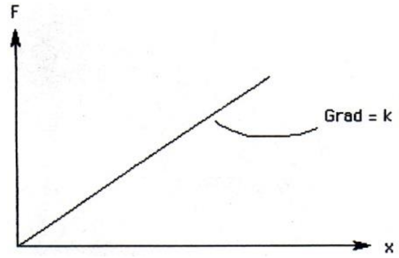
Energy stored = potential energy of spring = work done on spring

In about 1675 Robert Hooke noticed that the more you stretch a spring from it's natural length, the stronger the force needed.
i.e. $F \propto \Delta x$

we write $\mathbf{F = k\,x}$ Hooke's law

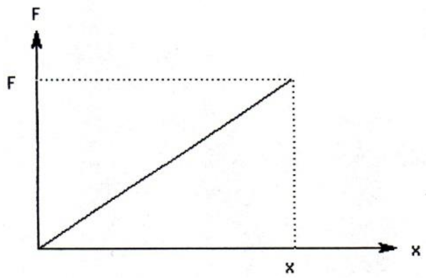
where k = spring constant (unit $N\ m^{-1}$)

we get a graph which looks like



Now P.E. of spring = Work done on it

We can calculate the work done on a spring in stretching it x metre from the force-distance graph.



Work done = area under graph (can't use $w = f \times x$ because force not constant)
= $\frac{1}{2} F x$

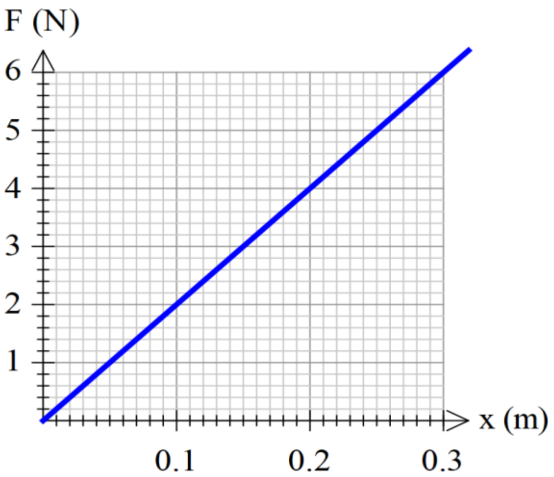
But F = k x

So work done = ½ k x x
= ½ k x^2

∴ **U_s = ½ k x^2 (Joule)**

Note: For a spring compressed and then released Δ P.E. (spring) = Δ K.E. (body).
Conservation of energy. $E_{total} = E_K + U_s$

Example 1. Find the P.E. of the spring when compressed 0.2 m.



$U_s = area$

$U_s = \frac{1}{2} \times 0.2 \times 4$

$U_s = 0.4\ J$

Example 2.
For a spring with k = 5 N m^{-1}. Find

a) Δ P.E. when compressed from 0→20 cm

U_s = ½ k x^2
U_s = ½ × 5 × 0.20^2
U_s = 0.1 J

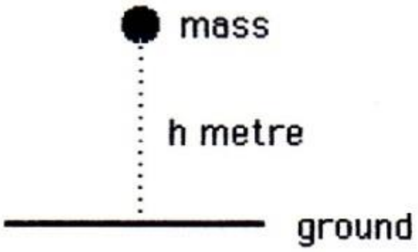
b) If compressed by 20 cm and a body is placed there and let go. What is the K.E. as it passes zero compression?

$U_s \rightarrow E_k$
$E_k = 0.1\ J$

M.7.5.2.2 Gravitational Potential Energy

When an object is raised above the surface of the Earth energy is stored.
To raise a body above the ground we must do work against the weight force.

Let us raise a mass, m, h metre above the ground



U_g = work done against weight force
$= F \times x$
$= m\,g\,h$ (Joule)

$\therefore$ **U_g = m g h (Joule)**

Examples

1) A mass of 5 Kg is raised 6 m above the ground. What is it's P.E.?

$U_g = m\,g\,h$
$U_g = 5 \times 9.8 \times 6$
$U_g = 294$ J

2) A mass of 3 Kg is 7 m above the ground. If it is released, what is it's K.E. just before it hits the ground? What is it's speed?

$U_g = m\,g\,h$
$U_g = 3 \times 9.8 \times 7$
$U_g = 205.8$ J

$U_g \rightarrow E_k$
$E_k = 205.8\,J$

$E_K = ½\,m\,v^2$
$205.8 = ½ \times 3 \times v^2$
$137.2 = v^2$
$v = 11.7$ m/s

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M.7.6 Power (mechanical)

The rate at which work is done on, or by a body is called power.

Power = $\frac{\text{Work done}}{\text{Time taken}}$

or $P = \frac{w}{t} = \frac{\Delta E}{t}$

Units of power are joule per second = Watts (W)

Example
The fastest woman to scale the Rialto building stairs in the Great Rialto Stair Trek, in a particular year climbed the 1222 steps, which are a total of 247 m high, in 7 min 58 s. Given that her mass is 60 kg, at what rate was she using energy to overcome the gravitational force alone?

$$P = \frac{\Delta E}{t}$$
$$\Delta E = U_g = m\ g\ h = 60 \times 9.8 \times 247 = 145236\ J$$
$$t = (7 \times 60) + 58 = 478$$
$$P = \frac{\Delta E}{t}$$
$$P = \frac{145236}{478}$$
$$P = 303.8\ W$$

M.7.6.1 Efficiency

In the real world all of the energy is never transformed to the new type. The percentage that is transformed into what you want is called efficiency.

$$Efficiency\ (\eta) = \frac{useful\ energy\ out}{total\ energy\ in} \times 100\ \%$$

Example
An electric kettle uses 23.3 kJ of electrical energy as it boils water. The efficiency is 18%. How much of this energy is actually transferred to the water as heat?

$$Efficiency\ (\eta) = \frac{useful\ energy\ out}{total\ energy\ in} \times 100\ \%$$
$$18 = \frac{useful\ energy\ out}{23.3 \times 10^3} \times 100$$
$$useful\ energy\ out = \frac{18\ \times\ 23.3 \times 10^3}{100}$$
$$useful\ energy\ out = 4194\ J = 4.194\ kJ$$

M.7.6.2 Power force and Average Speed

In everyday situations friction is involved. A force is required to keep things moving at constant speed. In this case power can be calculated from force and speed.

$$P = \frac{Work}{time} \quad \text{and} \quad work = F\,x$$

so $$P = \frac{F\,x}{time} \quad \text{but} \quad v_{ave} = \frac{x}{t}$$

Thus $$P = F\,v_{ave}$$

Example
Calculate the power required to keep a car moving at an average speed of 22 m/s if the force of friction is 1200 N.

$$P = F\,v_{ave}$$
$$P = 1200 \times 22$$
$$P = 26400 = 26.4\ kW$$

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(FIFTH TEST AT THIS POINT)

Revision: Text Page 443 All Questions