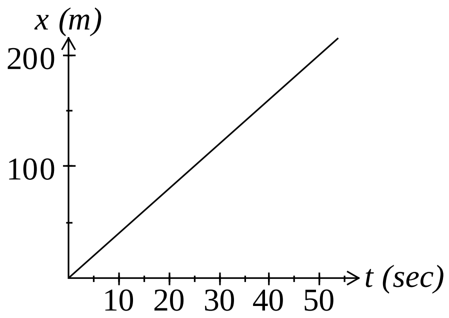
Physics with Synno – Motion-2 – Lesson 7

M.2.5 Graphs of x, v, a and t

Consider the position-time graph of a person who walks 200 m to the shop at a uniform rate in 50 seconds.



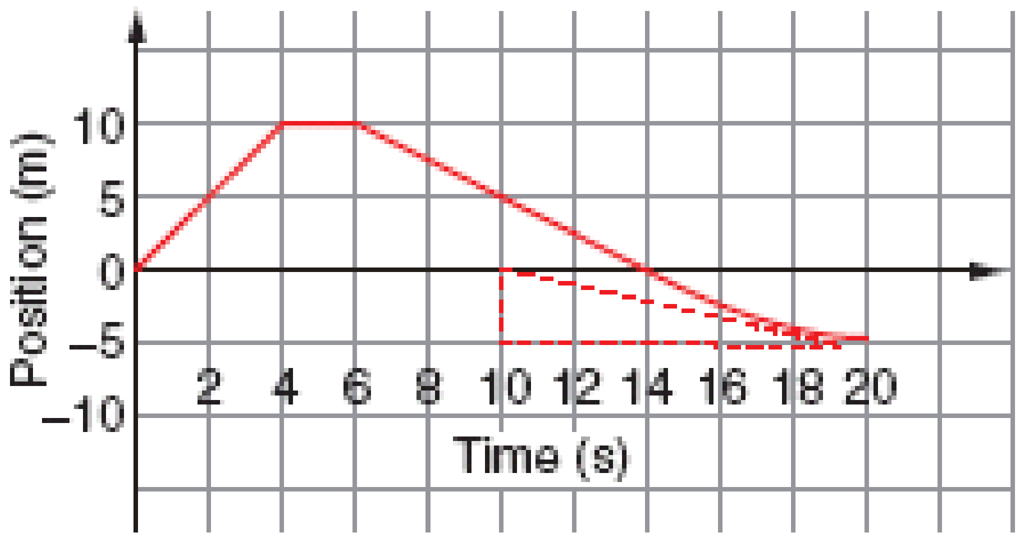
The gradient of a graph is $\frac{rise}{run}$

In this case the gradient is $\frac{\Delta x}{t}$ which is the velocity (speed), which in this case is 4 ms^{-1}.

Thus the gradient of an x-t graph is the velocity.

Example

A car driven by a learner driver travels along a straight driveway and is initially heading north. The position of the car is shown in the graph



a Describe the general motion of the car.

Something along the lines of

The car travels north at constant velocity for 10m, it then stops for 2 seconds, it then reverses at constant velocity for 10 seconds back to its starting point. It then slows to a stop over a 6 second period, 5m south of its original position.

b What is the displacement of the car during the first 10 s of its motion?

Displacement = final position – initial position = 5m North

c What distance has the car travelled during the first 10 s?

Distance = 10m + 5m = 15m

d Calculate the average velocity of the car during the first 4 s.

$$\vec{v} = \frac{displacement}{time} = \frac{10}{4} = 2.5 \text{ m/s North}$$

e Calculate the average velocity of the car between *t* = 6 s and *t* = 20 s.

$$\vec{v} = \frac{displacement}{time} = \frac{-5-10}{20-6} = \frac{-15}{14} = -1.07 \text{ m/s} = 1.07 \text{ m/s South}$$

f Calculate the average velocity of the car during its 20 s trip.

$$\vec{v} = \frac{displacement}{time} = \frac{-5-0}{20} = \frac{-5}{20} = -0.25 \text{ m/s} = 0.25 \text{ m/s South}$$

g Calculate the average speed of the car during its 20 s trip.

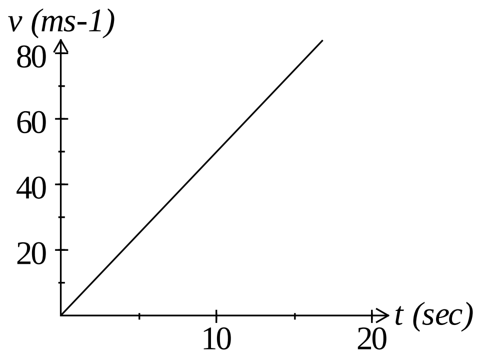
$$speed = \frac{distance}{time} = \frac{10+10+5}{20} = \frac{25}{20} = 1.25 \text{ m/s}$$

h Calculate the instantaneous velocity of the car at *t* = 18 s.

The gradient of an x-t graph is the velocity. Using the triangle drawn on the graph.

$$\vec{v}_{inst} = \frac{Rise}{run} = \frac{-5}{9} = -0.56 \text{ m/s} = 0.56 \text{ m/s South}$$

Now look at the following v-t graph.



The gradient of a graph is $\frac{rise}{run}$ which is $\frac{\Delta v}{t}$ which is the acceleration. In this case it is 4 ms^{-1}.

Thus the gradient of a v-t graph is the acceleration.

Also we know that $v = \frac{\Delta x}{t}$

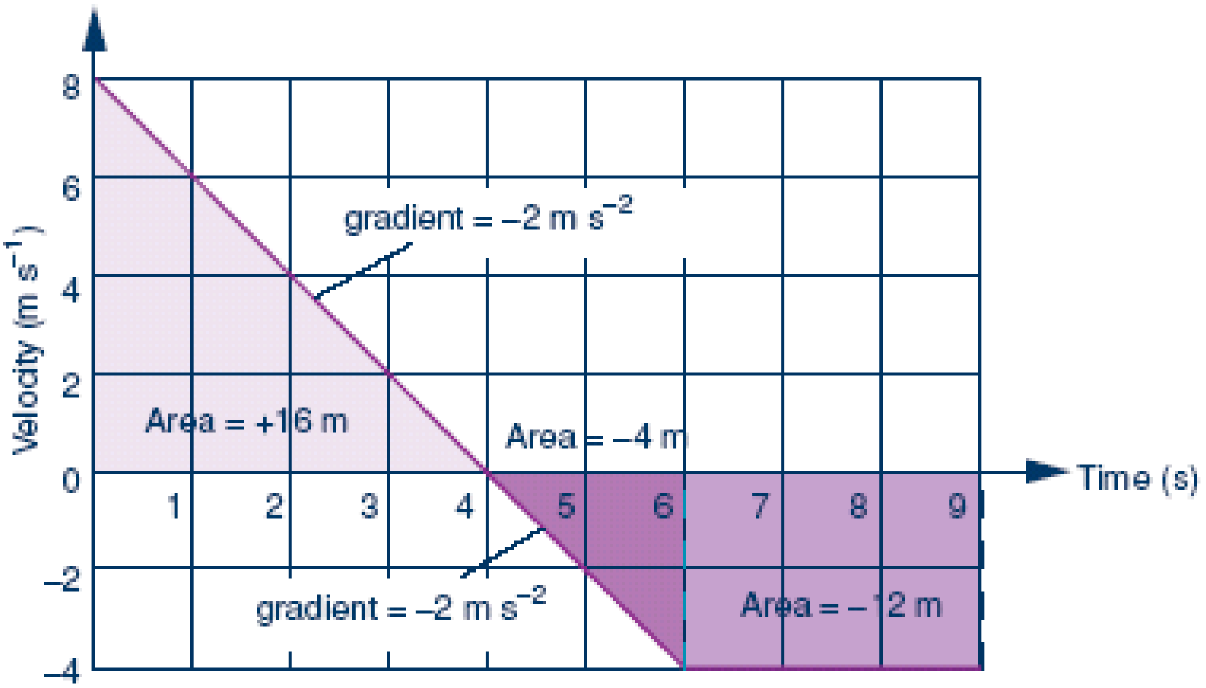
∴ $\Delta x = v\, \Delta t$

but v Δt is the area under the graph.

Thus the area under a v-t graph is equal to (Δx) the change in position (displacement).

Example

The motion of a marble rolling across a floor is represented by the graph



Use this graph to help you to:

a describe the motion of the marble

Something along the lines of
The marble starts with a velocity of 8 m/s it accelerates uniformly (Note: slowing negative acceleration) to be momentarily at rest after 4 seconds. It continues to accelerate at the same uniform rate for another 2 second where it reaches a velocity of - 4 m/s, which it maintains for another 3 seconds.

b calculate the displacement of the marble during the first 4 s

Thus the area under a v-t graph is equal to (Δx) the change in position (displacement).

$$displacement = area = \frac{b \times h}{2} = \frac{8 \times 4}{2} = 16\ m$$

c determine the displacement for the 9 s shown

$$displacement = area\ 0 - 4\,\text{sec} + area\ 4 - 6\,\text{sec} + area\ 6 - 9\ sec$$
$$= \frac{8 \times 4}{2} + \frac{2 \times -4}{2} + 3 \times -4 = 16 + -2 + -12 = 2\ m$$

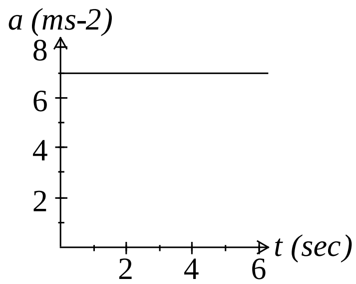
d find the acceleration during the first 4 s

$\vec{a} = \frac{\Delta v}{t} = \frac{0 - 8}{4} = -2\ \text{m/s}^2$

e find the acceleration from 4 s to 6 s.

$\vec{a} = \frac{\Delta v}{t} = \frac{-4 - 0}{2} = -2\ \text{m/s}^2$

Now consider what happens when you have an a-t graph and want to find out velocity.



We know $\vec{a} = \frac{\Delta v}{t}$

∴ $\Delta v = a \times t$

but $a \times t$ is the area under the graph.

Thus the area under an a-t graph is equal to (Δv) the change in velocity (speed).

Summary of Graphs

Graph	Gradient	Area	Read Directly
x-t	Velocity	-	Position at any time
v-t	Acceleration	Displacement	Velocity at any time
a-t	-	Change in Velocity	Acceleration at any time

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