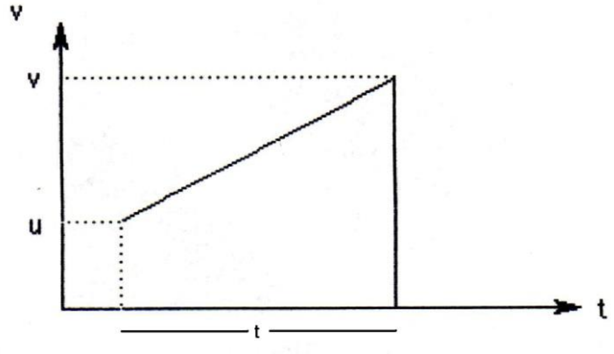
Physics with Synno – Motion-2 – Lesson 8

M.2.6 Constant Acceleration Formulae

We know the shape of a v-t graph when the acceleration is constant. It is a straight line as shown.



Definitions

a = acceleration
x = displacement
v = final velocity
u = initial velocity
t = time period

By taking the gradient of the v-t graph, we can obtain an expression for the acceleration.

$$\vec{a} = \frac{\Delta v}{t} = \frac{v-u}{t}$$

$\therefore$ v - u = a t

or v = u + a t Formula 1

By finding the area under the graph, we obtain an expression for the displacement. This can be done in two ways:

1. Breaking the area into a rectangle and a triangle.

   x = Area of rectangle + Area of triangle

   x = u t + ½(v -u)t

Substituting formula 1

x = u t + ½(u + a t - u)t
= u t + ½(a t)t

So $x = \text{u t} + \frac{1}{2}\text{a t}^2$ Formula 2

2. Using the formula for a trapezium. Area = ½(b + c)h.
In our case b = u, c = v, h = t

So $x = \frac{1}{2}(\text{u} + \text{v})\text{t}$ Formula 3

If we rearrange formula 1 we get $\text{t} = \frac{v-u}{a}$

Substituting this into formula 3 we get:

$$x = \tfrac{1}{2}(u + v)\left(\frac{v-u}{a}\right)$$

$$2ax = (u + v)(v - u)$$

$$2ax = u\,v - u^2 + v^2 - u\,v$$

$$2ax = v^2 - u^2$$

So $v^2 = u^2 + 2ax$ Formula 4

There is a fifth formula which can be derived

$$x = vt - \frac{1}{2}at^2$$

These formulae are known as the constant acceleration formulae.

The formulae	Not included
$v = u + at$	x
$x = \frac{1}{2}(u + v)t$	a
$x = ut + \frac{1}{2}at^2$	v
$x = vt - \frac{1}{2}at^2$	u
$v^2 = u^2 + 2ax$	t

Each equation uses 4 of the 5 different variables. Each equation uses a different combination of the 4 variables.

When solving problems using these equations, it is important that you think about the problem and try to visualise what is happening. The following steps are advisable.

Step 1 Draw a simple diagram of the situation.

Step 2 Neatly write down the information that has been given in the question, using positive and negative values to indicate directions. Convert all units to SI form (m, ms^{-1}, ms^{-2}, s).

Step 3 Select the equation that matches your data.

Step 4 Use the appropriate number of significant figures in your answer. Ie the number of figures in the answer must be the same as the number of figures in the information. (your answer cannot be more accurate than the information)

Step 5 Include units with the answer and specify a direction if the quantity is a vector.

Example.

A bobsled is sliding down a hill with an acceleration of 3 m/s^2. It starts from rest, if we ignore air resistance, find:

a) the speed after 5 seconds

$a = 3 \quad u = 0 \quad t = 5 \quad v = ?$
$v = u + a\,t$
$v = 0 + 3 \times 5 = 15$ m/s

b) how far it has travelled in 5 seconds

$a = 3 \quad u = 0 \quad t = 5 \quad x = ?$
$x = ut + \frac{1}{2}\,a\,t^2$
$x = 0 \times 5 + \frac{1}{2} \times 3 \times 5^2 = 37.5\,m$

c) the time taken to reach 30 m/s

$a = 3 \quad u = 0 \quad t = ? \quad v = 30$
$v = u + a\,t$
$30 = 0 + 3 \times t$
$t = 10\,sec$

d) the distance travelled when it reaches 45 m/s

$a = 3 \quad u = 0 \quad x = ? \quad v = 45$
$v^2 = u^2 + 2\,a\,x$
$45^2 = 0^2 + 2 \times 3 \times x$
$x = 337.5\,m$

Problem Set #8: Text Page 319 All Questions

(SECOND TEST AT THIS POINT)

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