G.1.2.2 Gravitational Potential Energy

Previously we have looked at small changes in the height of an object above the ground. In this case $\vec{g}$ is effectively constant and we use,

$$E_g = m\ \vec{g}\ \Delta h$$

However as we saw in section F.1.1, Newton's Law of Universal Gravitation tells us that $\vec{g}$ is not constant but varies inversely with the distance of separation squared.
The force distance graph for a small mass (m) as it moves away from a large mass (M) is shown below:

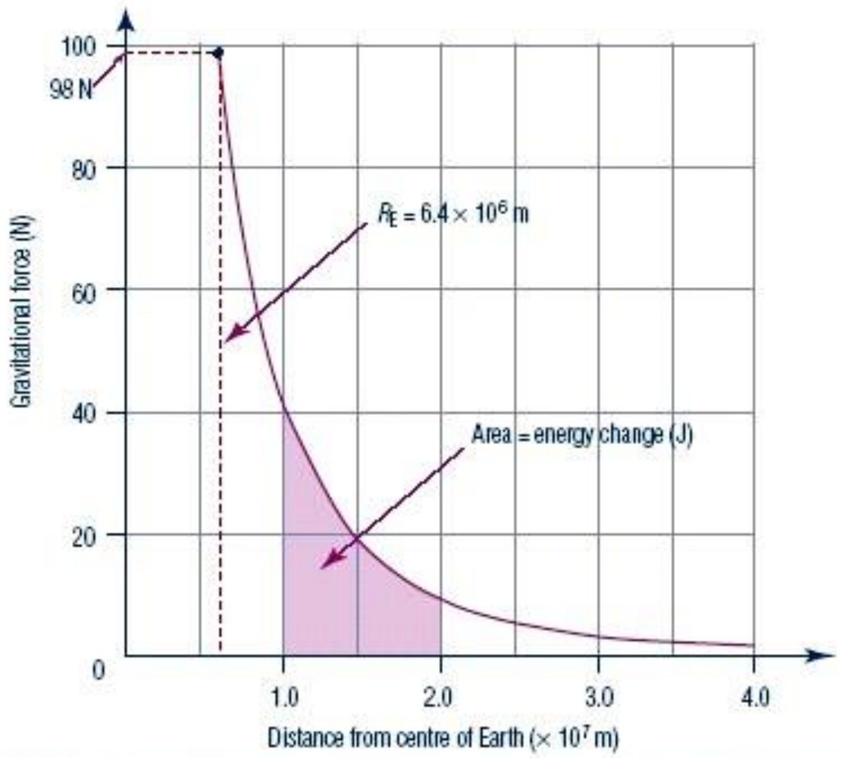
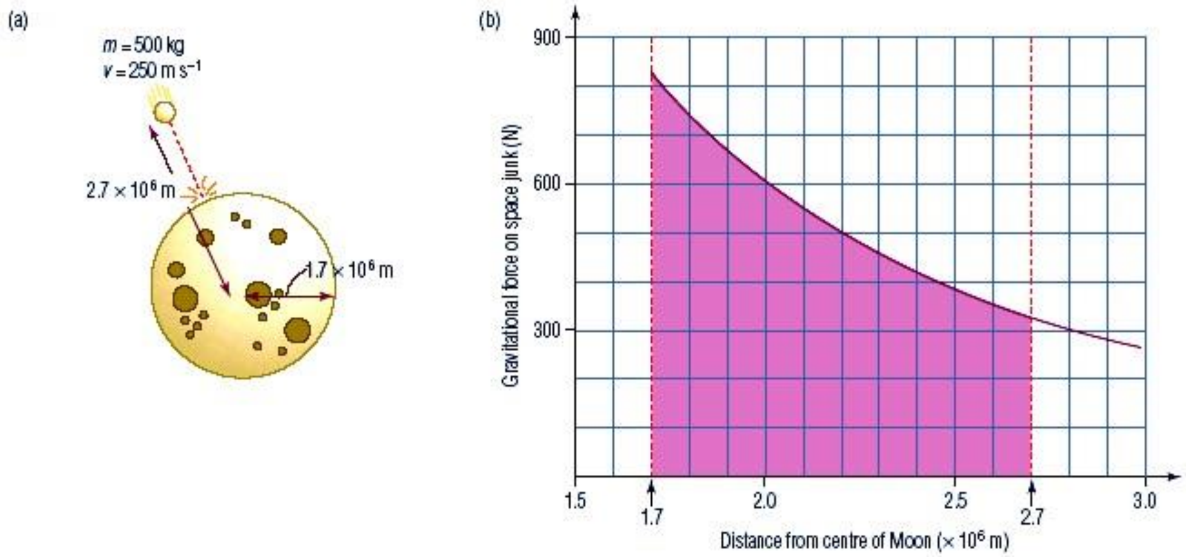


Figure 3.23 The gravitational force acting on a 10 kg body at different distances from the Earth. The shaded region represents the work done by the gravitational field as the body moves between 2.0×10^7 m and 1.0×10^7 m from the centre of the Earth.

As we know work done = change in energy = area under the graph.
Work done by gravitational field = area under force–distance graph = $\Delta E_k = -\Delta E_g$
Gravitational force–distance graphs are curved and so the usual method of determining the area under them is by using a counting squares technique.

Example

A 500 kg lump of space junk is plummeting towards the Moon as shown below. Its speed when it is 2.7×10^6 m from the centre of the Moon is 250 m s^{-1}. The Moon has a radius of 1.7×10^6 m.



Using the gravitational force–distance graph above, determine:

a the initial kinetic energy of the junk

b the increase in kinetic energy of the junk as it falls to the Moon's surface

c the speed of the junk as it crashes into the Moon

d the gravitational field strength at an altitude of 500 km.

Solution

a Initial kinetic energy of junk:

$$E_k = \frac{1}{2} m v^2$$

$$E_k = \frac{1}{2} \times 500 \times 250^2$$

$$E_k = 1.6 \times 10^7 \ J$$

b To find the increase in kinetic energy of the junk as it falls from a distance of 2.7×10^6 m to the surface, it is necessary to estimate the area under the force–distance graph between 2.7×10^6 m and 1.7×10^6 m from the Moon's centre. There are approximately 53 squares under this part of the graph. Each square represents 100 N $\times\ 0.1 \times 10^6$ m = 1.0×10^7 J.
Thus the increase in kinetic energy of the junk as it falls is:
$53 \times 1.0 \times 10^7 = 5.3 \times 10^8$ J

c The space junk has 1.6×10^7 J of kinetic energy initially, and gains 5.3×10^8 J as it falls. Therefore its kinetic energy as it strikes the Moon is:

$$E_k = 1.6 \times 10^7 + 5.3 \times 10^8$$

$$E_k = 5.46 \times 10^8 \text{ J}$$

To find the speed of the junk as it reaches the Moon's surface:

$$E_k = \frac{1}{2}mv^2$$

$$E_k = 5.46 \times 10^8$$

$$\Rightarrow \qquad v = \sqrt{\frac{5.46 \times 10^8}{0.5 \times 500}}$$

$$v = 1480 \text{ m s}^{-1}.$$

d 500 km is equal to 5.0×10^5 m.
This is a distance from the centre of the Moon of $1.7 \times 10^6 + 5.0 \times 10^5 = 2.2 \times 10^6$ m.
At this distance, the gravitational force on the 500 kg body is (reading from the graph) equal to 500 N. Thus the gravitational field strength at this altitude is:

$$\vec{g} = \frac{\vec{F}}{m}$$

$$\vec{g} = \frac{500}{500}$$

$$\vec{g} = 1.0 \text{ N kg}^{-1}$$

Text Questions: Pages 26 & 27 All Questions