Physics with Synno – Move Without Contact – Lesson 3

G.1.2.3 Satellites

A satellite is defined as a smaller object orbiting around a larger one. Satellites can be natural (such as the moon) or artificial (such as communications satellites). In all cases the satellite is held in orbit only by gravitational forces.

G.1.2.4 Circular Motion and Gravity

Let us consider a satellite orbiting the Earth. The path of the orbit will be almost circular. So we use the circular motion formulae to make calculations.

Thus $speed\,(v) = \frac{distance}{time} = \frac{2\pi r}{T}$ applies

The acceleration is: $\vec{a} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = \vec{g}$

The force is: $\vec{F} = \frac{mv^2}{r} = \frac{4\pi^2 r m}{T^2} = \frac{GMm}{r^2} = m\vec{g}$

Example.

Optus B is a geostationary satellite. Its period of orbit is 24 hours so that it revolves at the same rate at which the Earth turns. Given that the mass of the Earth is 6.0×10^{24} kg and the mass of Optus B is 1500 kg, calculate:
a its orbital radius
b the gravitational field strength at this radius
c Optus B's orbital speed
d its acceleration.

Solution

a Period T = 24 hours
T = 24 × 60 × 60
T = 86 400 s
To calculate R, using acceleration due to gravity:

$$\vec{g} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{6.67\times10^{-11} \times 6.0\times10^{24} \times 86400^2}{4\pi^2}}$$

$r = 4.23 \times 10^7$ m
So the orbital radius of Optus B is 42 300 km.

b Gravitational field strength:

$$\vec{g} = \frac{GM}{r^2}$$

$$\vec{g} = \frac{6.67\times10^{-11} \times 6.0\times10^{24}}{\left(4.23\times10^{7}\right)^2}$$

$$\vec{g} = 0.22\ N\,kg^{-1}$$

c The speed of Optus B is given by:

$$v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi \times 4.23\times10^{7}}{86400}$$

$$v = 3.08 \times 10^{3}\ \text{m s}^{-1}$$

d The acceleration of the satellite as it orbits is equal to the gravitational field strength at this radius as calculated in part b, i.e. $a = g = 0.22\ \text{m s}^{-2}$.

G.1.2.5 Kepler's Laws

Video: Kepler's Laws of Planetary Motion

In 1609, Kepler, a German astronomer, published his three laws on the motion of planets.

1. The planets move in elliptical orbits with the Sun at one focus.
2. The line connecting a planet to the Sun sweeps out equal areas in equal times
3. For every planet the ratio of the cube of the average orbital radius to the square of the period of

   revolution is the same, i.e. $\frac{R^3}{T^2}$ = constant

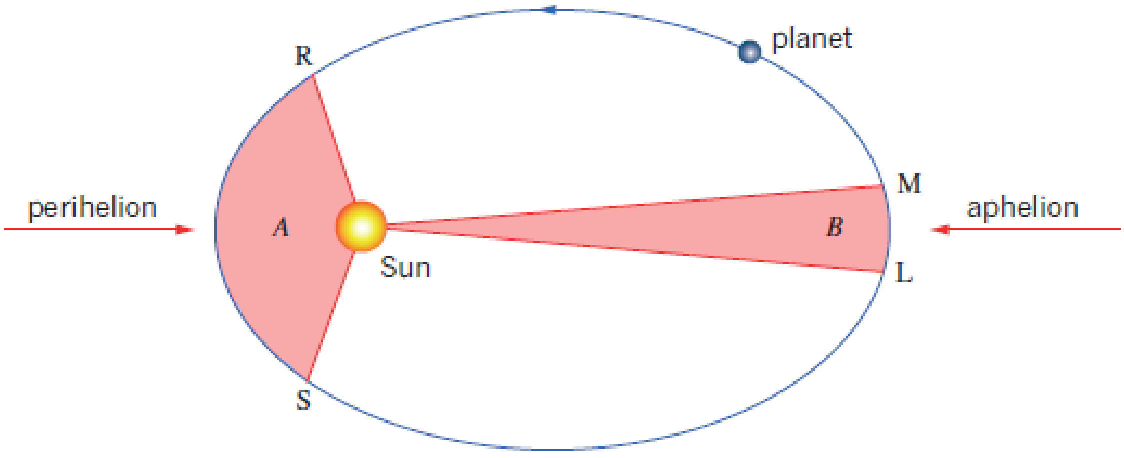


FIGURE 3.1.20 The planets, which are natural satellites of the Sun, orbit in elliptical paths with the Sun at one focus. Their speeds vary continually, and they are fastest when closest to the Sun. A line joining a planet to the Sun will sweep out equal areas in equal times. So, for example, the time it takes to move from *R* to *S* is equal to the time it takes to move from *L* to *M*, and so area *A* is the same as area *B*.

The third idea has been frequently used in the past on exams. It is derived from

$\therefore \qquad F = \dfrac{GMm}{R^2}$

$\therefore \qquad$ and $\dfrac{mv^2}{R} = \dfrac{4\pi^2 Rm}{T^2}$

$\therefore \qquad \dfrac{G\,M\,m}{R^2} = \dfrac{4\,\pi^2 R\,m}{T^2}$

Rearranging, gives

$\therefore \qquad \dfrac{G\,M}{4\,\pi^2} = \dfrac{R^3}{T^2}$

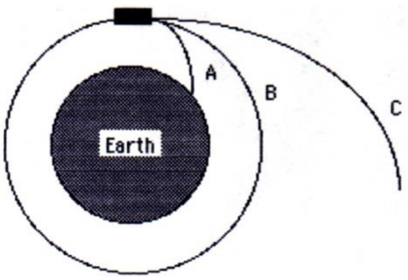
This means that for any given central body of mass M, the ratio $\dfrac{R^3}{T^2}$ = constant.

Therefore the constant = $\dfrac{GM}{4\pi^2}$ for all its satellites.

Note that this is <u>not</u> a universal constant, but one that is constant for each central body, i.e. It will be different for the moon orbiting the earth and the earth orbiting the sun.

G.1.2.6 The Speed of a Satellite

For a satellite to orbit the Earth it must have some minimum velocity, otherwise it will fall down to Earth.



A too slow, C too fast, B just right.

Looking at the force on the satellite.

The centripetal force = The force of gravity

$$\frac{mv^2}{r} = \frac{G\,M\,m}{r^2}$$

$$v^2 = \frac{G\,M}{r}$$

$$\therefore \qquad v = \sqrt{\frac{G\,M}{r}}$$

where M is the mass of the central body.
r is the radius of the orbit.

Example

Ganymede is the largest of Jupiter's moons. It has a mass of 1.66×10^{23} kg, an orbital radius of 1.07×10^{6} km and an orbital period of 6.18×10^{5} s (7.15 days).

a Use Kepler's third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.

$$\frac{R^3}{T^2} = \text{constant.}$$

b Use the orbital data for Ganymede to calculate the mass of Jupiter.

$$\frac{G\,M\,m}{R^2} = \frac{4\,\pi^2 R\,m}{T^2}$$

c Calculate the orbital speed of Ganymede in km s^{-1}.

$$v = \frac{2\,\pi\,r}{T}$$

Text Questions: Page 87 Questions 6 – 10