Physics with Synno – Light-Matter – Lesson 2

L&M.2.1 Einstein's Explanation

Einstein wrote a paper to explain the photoelectric effect for which he received the 1921 Nobel Prize. Einstein's explanation of these results was that the energy of light is not spread evenly across a uniform wave front but concentrated in separate 'lumps' or packets of energy. The energy of these packets has a definite value that depends on the frequency of light radiation. A more intense light source contains more packets but each packet still has the same amount of energy.
This explains the immediate ejection of electrons even for very weak light, because if the photon has enough energy it can be absorbed and the electron is ejected. However if the frequency is too low, each photon will not possess enough energy for the electron to be ejected, no matter how many photons are present (i.e. how bright the light is made).

Extrapolating the previous graph:

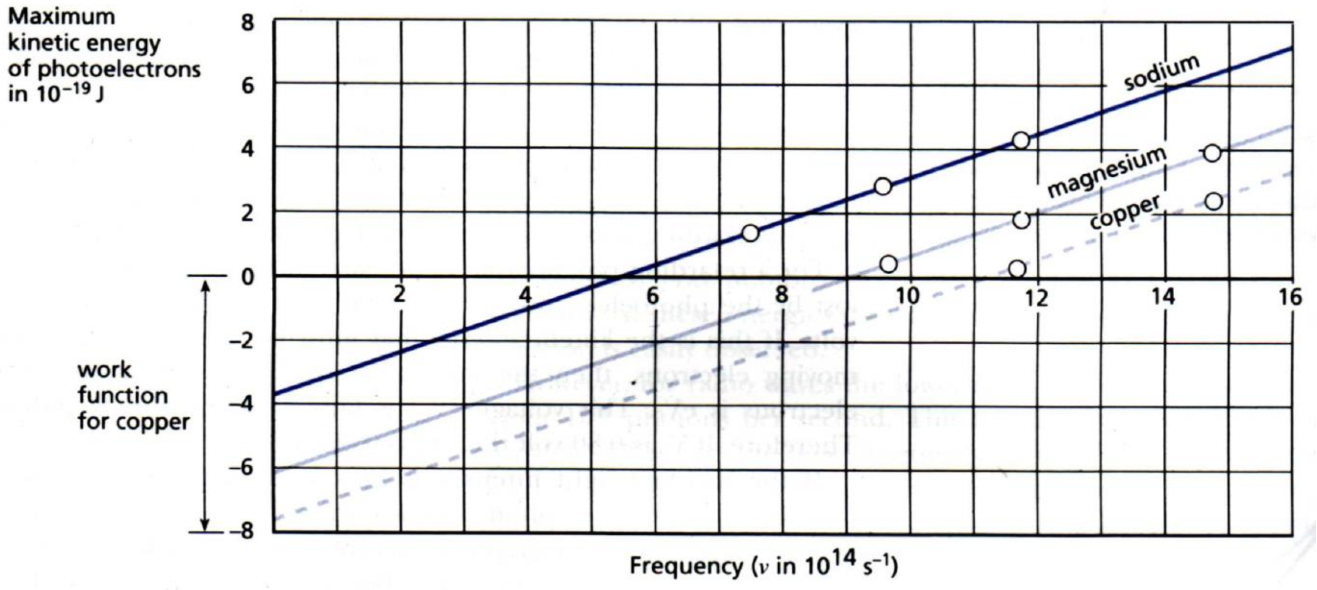

Figure 20.9
The maximum kinetic energy of the ejected photoelectrons plotted against the frequency of the incident radiation. Each graph has the same slope. The slope is Planck's constant.

The equation of this line is $K.E._{max} = hf - W$

hf is the quantity of energy delivered to the photoelectron by the photon.
However the electron loses some of this energy on the way to the metal's surface through collisions and some energy is required for the electron to escape the surface.
This work done by the electron in escaping the surface is W.
The excess energy $hf - W$ will be the kinetic energy that the photoelectron escapes with.

Thus $KE_{max} = hf - W$.

Note:

1. W is the minimum amount of work done in escaping the metal's surface and is called the work function.

2. The work function W is characteristic of the cathode material. $W = h f_o$

3. Energy is often measured in electron volts (eV). $1 \text{ eV} = 1.60 \times 10^{-19}$ J

Example

Yellow–green light of wavelength 500 nm shines on a metal whose stopping voltage is found to be 0.80 V. The mass of an electron is 9.11×10^{-31} kg. Find: the work function of the metal in both joules and electron volts.

Solution

$KE_{max} = h f - W$
$W = h f - KE_{max}$
$W = \frac{h\,c}{\lambda} - e\,V_0$
$W = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{500 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.8$
$W = 2.69 \times 10^{-19}$ J or 1.68 eV

Example :

Ultra violet light of wavelength 200 nm is incident on a clean silver surface. The work function of the silver is 4.7 eV. What is

a) The kinetic energy of the fastest-moving ejected electrons?

$KE_{max} = h f - W$ $\quad E_{photon} = h\,f$ also $\quad E = \frac{h\,c}{\lambda}$
$KE_{max} = \frac{h\,c}{\lambda} - W$
$KE_{max} = \frac{4.1 \times 10^{-15} \times 3.0 \times 10^{8}}{200 \times 10^{-9}} - 4.7$
$KE_{max} = 56.8\ eV$

b) The kinetic energy of the slowest-moving electrons?

Messing with your head on this one.
The slowest would be stopped
$KE = 0\ eV$

c) The cut-off frequency (f_o) for silver?

$W = h f_o$
$4.7 = 4.1 \times 10^{-15} \times f_0$
$f_0 = 1.15 \times 10^{15}\ Hz$

d) The cut-off wavelength for silver?

$$v = \lambda f \qquad v = 3.0 \times 10^{8}\ for\ light$$
$$3.0 \times 10^{8} = \lambda \times 1.15 \times 10^{15}$$
$$\lambda = 2.61 \times 10^{-7}\ m$$
$$\lambda = 261\ nm$$

e) The cut-off potential V_o for silver?

$$E = q\,V \quad \text{for } V_0 = ? \quad use\ E = W\ (change\ into\ J)$$
$$E = W = 4.7 \times 1.6 \times 10^{-19} = 7.52 \times 10^{-19}\ J$$
$$7.52 \times 10^{-19} = 1.6 \times 10^{-19} \times V_0$$
$$V_0 = 4.7\ V$$

This is the same value as the work function, this is not a coincidence

L&M.3 The Momentum of a Photon

Since the velocity of a photon is that of the speed of light c. We can write the momentum as

$$p = m\,c$$

Using Einstein's famous equation E = mc^2 and E = h*f*. We get

$$m\,c^2 = h\,f$$

$$m = \frac{h\,f}{c^2}$$

So the momentum is

$$p = \frac{E}{c} = \frac{h\,f}{c} = \frac{h}{\lambda}$$ (since c = λ *f*)

Example :

What is the momentum of a photon of red light of wavelength 650 nm?

$$p = \frac{h}{\lambda}$$
$$p = \frac{6.63 \times 10^{-34}}{650 \times 10^{-9}}$$
$$p = 1.02 \times 10^{-27}\ kg\ m/s$$

L&M.4 Matter Waves

In 1923 the French Physicist, Prince Louis de Broglie postulated that all moving particles have waves associated with them. According to de Broglie, all moving particles exhibit both wave and particle properties. The waves associated with these particles are called de Broglie waves.
Louis de Broglie also predicted some properties of these waves, including the wave length. He looked at the momentum of a photon and said that this equation would hold for all particles. If this is the case then we can work out the wavelength.

$$P = \frac{h}{\lambda} \qquad \text{and} \qquad P = mv$$

or $$\lambda = \frac{h}{P} = \frac{h}{m\,v}$$

Thus the wavelength for any particle can be calculated. If this is done then we find that large objects have an extremely small wavelength that cannot be detected.

i.e. a 1 Kg mass travelling at 30 m/s

$$\lambda = \frac{6.63\times10^{-34}}{1\times30}$$

$$\lambda = 2..2\times10^{-35}\ m$$

Smaller objects such as electrons have wavelengths that can be detected.

i.e. an electron moving at 2×10^6 m/s

$$\lambda = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times2\times10^{6}}$$

$$\lambda = 3.6\times10^{-10}\ \text{m}$$

The spacing between atoms in a crystal structure is about the right size to observe diffraction.

L&M.5 Electrons as Waves

If de Broglie's theory was correct then electrons should behave as waves. Thus they should have the same behaviour as waves exhibiting diffraction and interference effects.

As we know to get significant diffraction the size of the slit and the wavelength need to be about the same $\left(\frac{\lambda}{w} \approx 1\right)$. It turns out that the distance between atoms in a crystal structure is the right size to observed diffraction patterns from electrons.

In 1927 Davisson and Germer in the USA and G.P. Thompson (son of J.J.) in the UK both performed experiments showing diffraction of electrons.

Davisson and Gerner were also able to prove de Broglie's prediction of the wavelength to be correct.

Davisson and Thompson shared the 1937 Nobel prize for their work in this area.

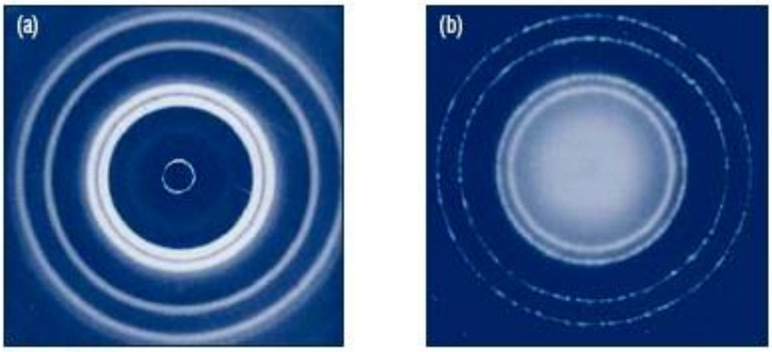

Figure 7.4 These diffraction patterns were taken by using (a) X-rays and (b) a beam of electrons with the same target crystal. Their similarity suggests a wave-like behaviour for the electrons.

However interference properties were still not able to be shown.
It was not until 1989 that a team of Japanese Physicists were able to show that single electrons exhibited an interference pattern.
In 1991 interference patterns for large particles, in particular Helium and Sodium, were shown.

Example

Figure 7.4 shows two images that have been obtained by scattering X-rays and electrons off a sample of many tiny crystals with random orientation. Assume that the X-rays used had a frequency of 8.6×10^{18} Hz and $c = 3.0 \times 10^8$ m/s

a **E**xplain why a 'circular' diffraction pattern is obtained, rather than a row of diffraction 'fringes' as studied earlier.

b Since the two diffraction patterns show exactly the same fringe spacing, what conclusions can be made about the electrons and X-rays that were used.

c Determine the de Broglie wavelength of the electrons.

d **C**ompare the momentum of one electron to the momentum of one X-ray photon.

Solution

a The ‘gap’ through which the X-rays and electrons are passing is not a vertical slit.

b The amount of diffraction depends on gap size and wavelength. The gap size is the same for both, thus the wavelength must also be the same.

c λ electrons = λ X-ray photons

$= \frac{c}{f}$

$= 3.0 \times 10^{8} \div 8.6 \times 10^{18}$

$= 3.5 \times 10^{-11}$ m

d For both photons and particles: $p = h\,\lambda$

Since the X-rays and electrons have equivalent ‘wavelengths’, they must have equivalent momentum.

L&M.6 What then is light?

The photoelectric effect suggests that light is a stream of energy 'packets' or particles. However if we direct a stream of these 'particles' at a double slit apparatus such that the photon intensity is so weak that less than one photon per second arrives at the slits we find that an interference pattern exactly like that predicted by the wave model is produced.

So, is light a particle or a wave? It displays properties of both and yet it cannot be both a wave and a particle. The solution to this dilemma lies in a complex and mathematically abstract subject called Quantum Mechanics. The details of which were worked out independently by Erwin Schrodinger (wave mechanics) and Werner Heisenberg (matrix mechanics) in 1933.

These models do not provide a physical picture in familiar concepts. So we must accept wave-particle duality and choose the model appropriate to the problem we are solving.

E.g.
Refraction
Diffraction
Interference — Wave model
Reflection
Intensity
Photoelectric effect — Particle (photon) model
Quantised energy levels

Text Questions: Page 345 Exercise 10.2 Questions 1 – 10