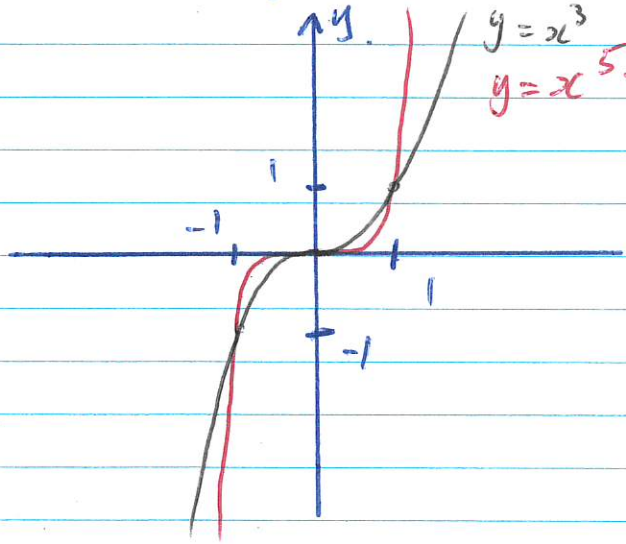
POWER FUNCTIONS.

A power function is of the form $y = x^n$
Here we will look at n being a positive integer

* n odd.

ie $f(x) = x^3$ $f(x) = x^5$, etc.



Common features
Pass through $(1, 1)$ and $(-1, -1)$.
Stationary Point of inflection at $(0, 0)$.
as n gets bigger - flatter around POI
- Steeper after $(1, 1)$ and $(-1, -1)$

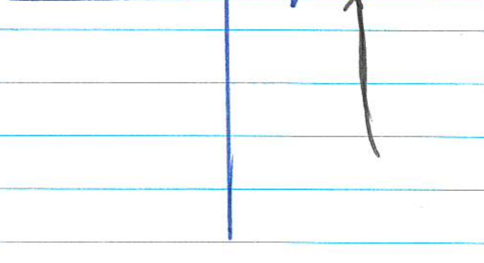
Transformation as per other graphs.

Eg Sketch the graph of $y = -(x-1)^3 + 2$.
* Reflection in x axis
* Translation - Right 1
- Up 2

$\Rightarrow$ P.O.I $(1, 2)$.

y-int $x = 0$
$y = -(0-1)^3 + 2$
$= 3$.

x-int $y = 0$
$0 = -(x-1)^3 + 2$.
$(x-1)^3 = 2$.
$x - 1 = \sqrt[3]{2}$.
$x = 1 + \sqrt[3]{2}$.



Eg Find the image of the graph of $y = x^5$ under the following transformations

- Reflection in y-axis
- Dilation factor 2 from y-axis (in x-dirⁿ)
- Translation −2 units +ve x direction, −3 units +ve y direction

Using Mapping

$$(x, y) \to (-x, y) \to (-2x, y) \to (-2x+2, y+3)$$

$x' = -2x + 2$

$x' - 2 = -2x$

$\dfrac{x' - 2}{-2} = x$

$y' = y + 3$

$y = y' - 3$

$$y' - 3 = \left(\frac{x' - 2}{-2}\right)^5$$

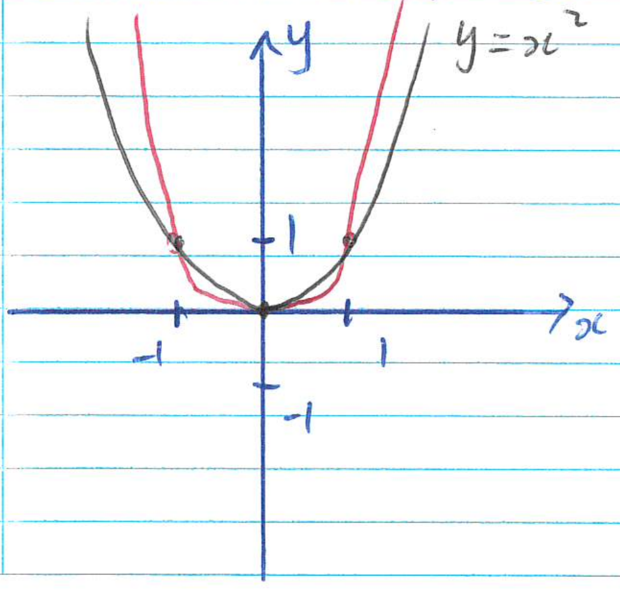
$$y - 3 = \frac{(x-2)^5}{(-2)^5}$$

$$y = \frac{(x-2)^5}{-32} + 3$$

OR $y = -\dfrac{1}{32}(x-2)^5 + 3$

* n even

ie $f(x) = x^2$ $f(x) = x^4$ etc.



Common Features

Pass through (1, 1) and (−1, 1)

Turning point at (0, 0).

As n gets bigger:

- Flatter around T.P.
- Steeper after (1, 1) and (−1, 1)

Eg The graph of $y = a(x-h)^4 + k$ has a turning point at $(2, 2)$ and passes through $(0, 4)$. Find a, h and k.

T.P. $(2, 2)$.

$$\rightarrow y = a(x-2)^4 + 2.$$

$(0, 4)$

$$4 = a(0-2)^4 + 2.$$
$$2 = a(-2)^4$$
$$2 = 16a$$
$$\frac{2}{16} = a$$
$$a = \frac{1}{8}$$

$$y = \frac{1}{8}(x-2)^4 + 2.$$

Exercise 3G Q^{ns} on Work Plan.