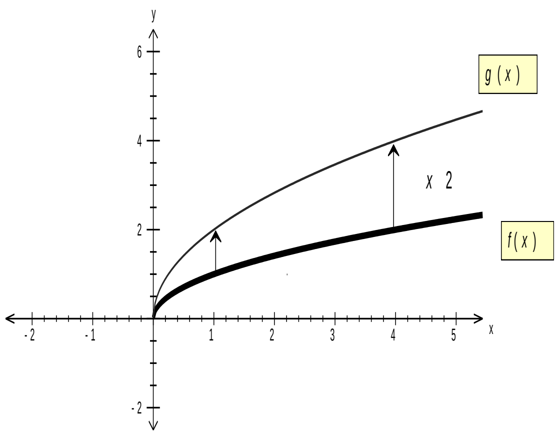
Transformations

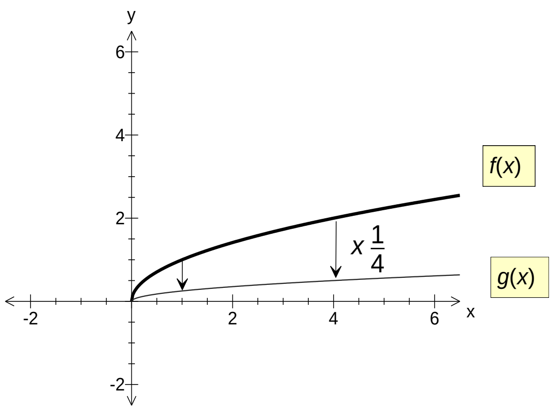
The graph of $y = x^2$ (or any other) can be changed so that its position or orientation are different, but still maintain the features of a parabola. When a graph is changed in this way it is said to be transformed. The transformations which relate to graphs are:

- ✓ Dilation
- ✓ Translation
- ✓ Reflection

1. Dilation:- A transformation where the graph is 'stretched' in a particular direction.

   a) Dilation from the *x*-axis





$f(x) = \sqrt{x}$
$g(x) = 2\sqrt{x}$

$f(x) = \sqrt{x}$
$g(x) = \frac{1}{4}\sqrt{x}$

In each of the examples above the graphs of $f(x)$ has been *transformed* into $g(x)$ via a *dilation* away from the $x-axis$.

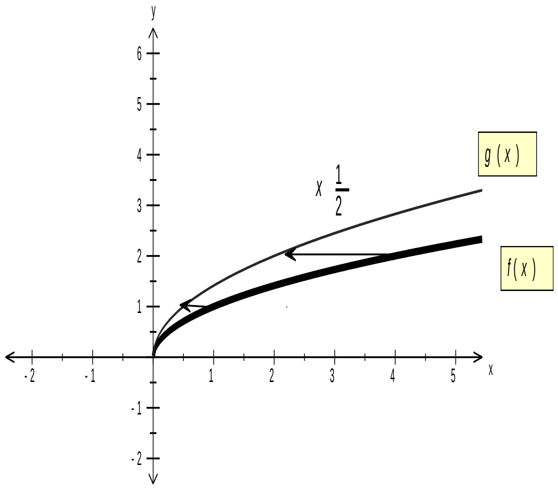
The left hand graph shows a *dilation* of *factor 2* away from the $x-axis$. Each of the $y-values$ of $f(x)$ has been multiplied by 2 to get $g(x)$.

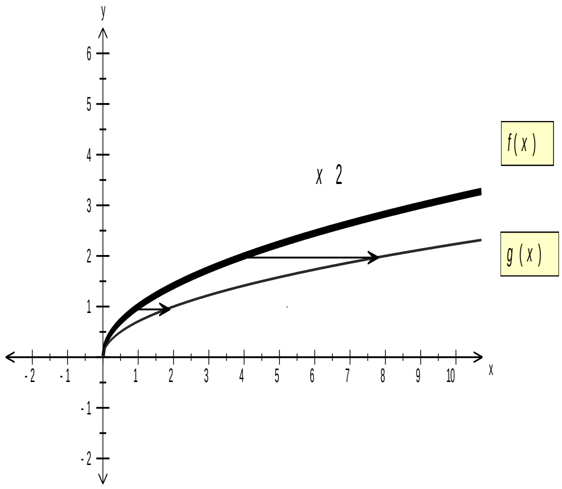
On the right hand graph the $y-values$ of $f(x)$ have been multiplied by $\frac{1}{4}$ to get $g(x)$. So this is referred to as a *dilation* of *factor* $\frac{1}{4}$ away from the $x-axis$.

The relationship between $g(x)$ and $f(x)$ can be written as $g(x) = a\,f(x)$, where *a* is the dilation factor.

Note: this can also be described as dilation parallel to the *y*-axis.

b) Dilation away from the $y-axis$.





$f(x) = \sqrt{x}$

$g(x) = \sqrt{2\,x}$

$f(x) = \sqrt{x}$

$g(x) = \sqrt{\frac{x}{2}}$

In each of the examples above the graphs of $f(x)$ has been *transformed* into $g(x)$ via a *dilation* away from the $y-axis$.

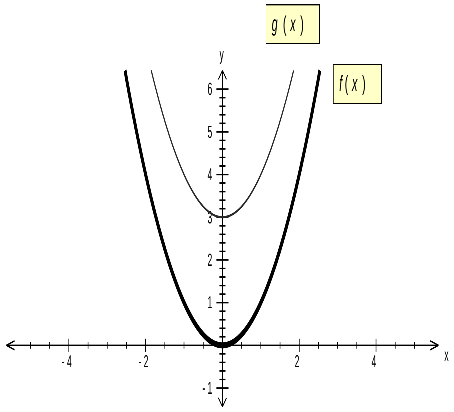
The left hand graph shows a *dilation* of *factor* $\frac{1}{2}$ away from the $x-axis$. Each of the $x-values$ of $f(x)$ has been multiplied by $\frac{1}{2}$ to get $g(x)$.

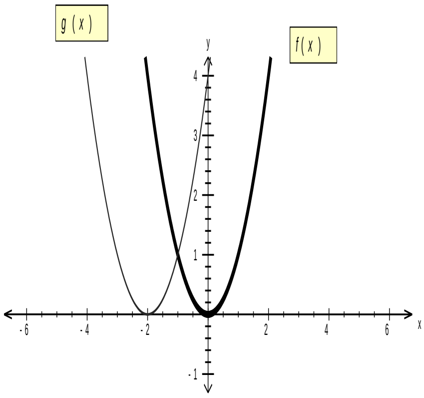
On the right hand graph the $x-values$ of $f(x)$ have been multiplied by 2 to get $g(x)$. So this is referred to as a *dilation* of *factor 2* away from the $x-axis$.

In general the relationship between $g(x)$ and $f(x)$ can be written as $g(x) = f(bx)$, where the dilation factor is $\frac{1}{b}$.

Note: this can also be described as dilation parallel to the *x*-axis.

2. Translation:- A transformation where the graphs position relative to the axes is changed, but not its orientation, size or shape.





$f(x) = x^2$
$g(x) = x^2 + 3$

$f(x) = x^2$
$g(x) = (x+2)^2$

In each of the examples above the graphs of $f(x)$ has been *transformed* into $g(x)$ via a *translation*.

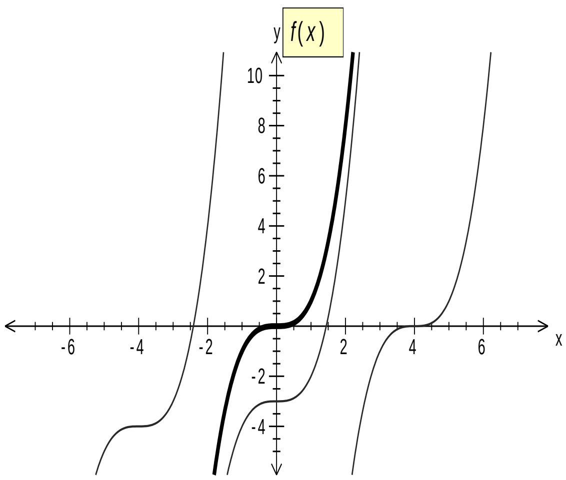
The left hand graph shows a *translation* of *3 units* parallel to the $y-axis$. Each of the $y-values$ of $f(x)$ has had 3 added to it to get $g(x)$. Sometimes this translation is describe a moving 3 unit up.

In general the relationship between $g(x)$ and $f(x)$ can be written as
$g(x) = f(x) + k$.

On the right hand graph the $x-values$ of $f(x)$ have had 2 added to them to get $g(x)$
So this is referred to as a *translation* of *–2 units* parallel to the $x-axis$. Sometimes this translation is referred to as moving 2 unit left.

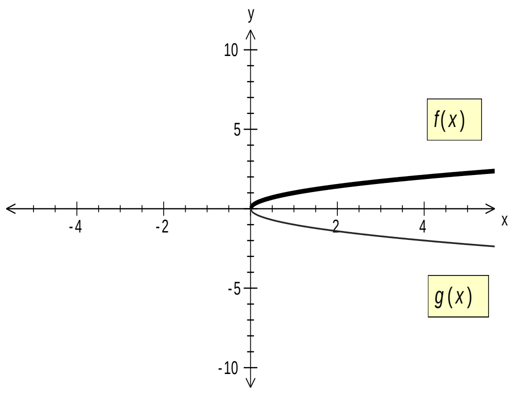
In general the relationship between $g(x)$ and $f(x)$ can be written as
$g(x) = f(x-h)$.

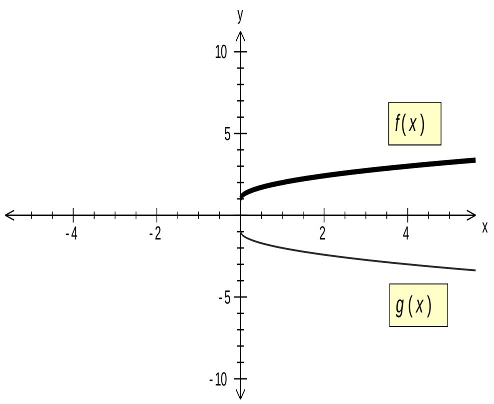
Graphs may also be translated down, to the right or by a combination.



3. Reflection:- A geometrical transformation of a graph from one side of a line to a symmetrical position on the other side. Typically, but not exclusively, graphs are reflected in the $x-axis$ or the $y-axis$.

   a) Reflection in the $x-axis$.





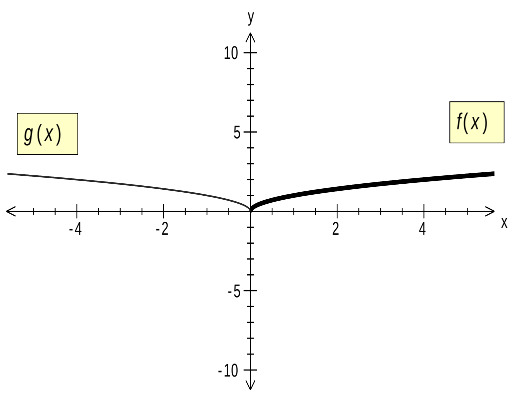
$f(x) = \sqrt{x}$
$g(x) = -\sqrt{x}$

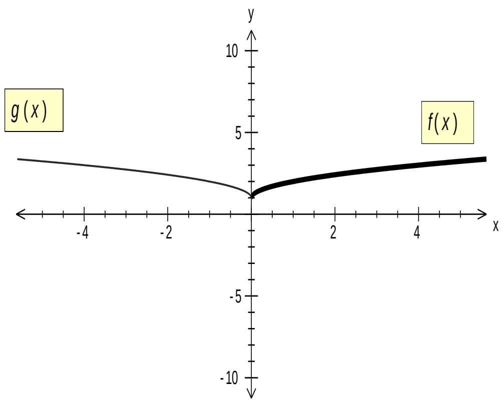
$f(x) = \sqrt{x} + 1$
$g(x) = -(\sqrt{x} + 1)$

In each of the examples above the graphs of $f(x)$ has been *transformed* into $g(x)$ via a *reflection* in the $x-axis$.

In general the relationship between $g(x)$ and $f(x)$ can be written as $g(x) = -f(x)$.

   b) Reflection in the $y-axis$





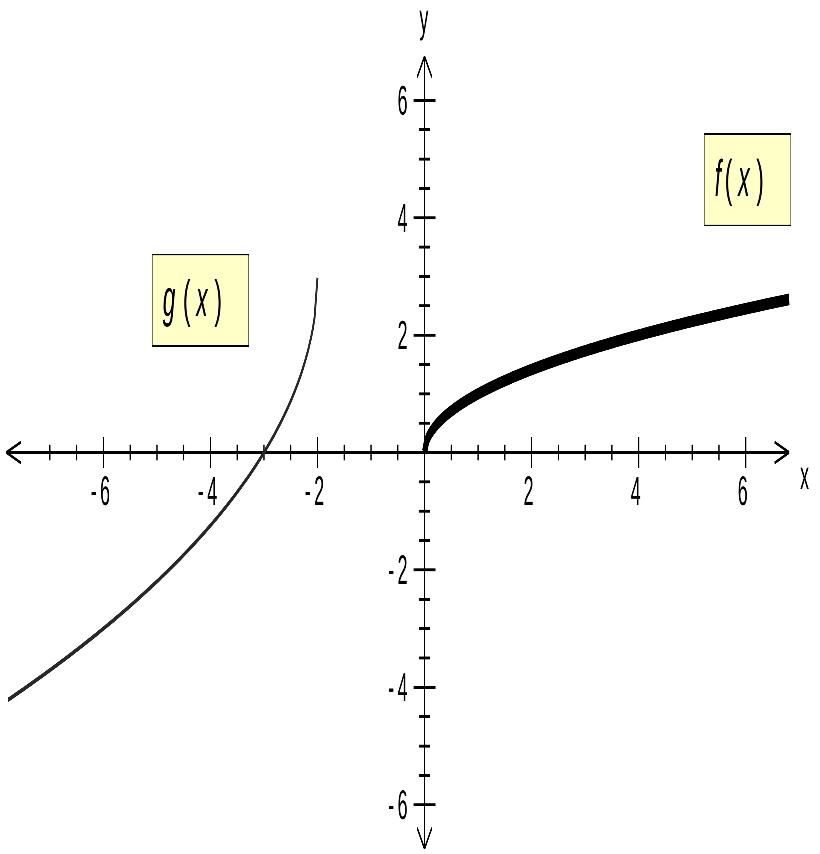
$f(x) = \sqrt{x}$
$g(x) = \sqrt{-x}$

$f(x) = \sqrt{x} + 1$
$g(x) = \sqrt{-x} + 1$

In each of the examples above the graphs of $f(x)$ has been *transformed* into $g(x)$ via a *reflection* in the $y-axis$.

In general the relationship between $g(x)$ and $f(x)$ can be written as $g(x) = f(-x)$.

Note: A graph may have any or all of these transformations applied to it.



$$f(x) = \sqrt{x}$$
$$g(x) = -3\sqrt{-(x+2)} + 3$$

In general the relationship between $g(x)$ and $f(x)$ can be written as

Reflection in *y*-axis

Reflection in *x*-axis

Translation in *y*-direction

$$g(x) = \pm a\, f(\pm\, b(x-h)\,) + k.$$

Dilation from *x-axis*

Translation in *x*-direction

Dilation from *y*-axis